Astronomy and Astrophysics

Lecture notes
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Contents

Preface

Each chapter consists of an **abstract** that summarizes briefly what we discuss in each chapter, followed by the main body and is usually closed by the learning goals of the corresponding section. The **colored boxes** are intended to provide an orientation for the reader:
Red boxes stand for key formulas and relations that you should definitely keep in mind:

Key formulas and relations

Green boxes summarize the content of the chapter:

Learning goals

they usually appear at the end of each section.
Thirdly, yellow boxes also give you a rough idea about possible exam content:

Important (Exam)

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Chapter 1

Classical Astronomy: Coordinates and Time

Abstract. Introduction of the different concepts of coordinate systems used for orbital & galactic navigation. Furthermore, problems of movement in this non-static environment as well as the correct measurements of distance and time are addressed.

Keywords: Coordinate system, coordinate transformation, apparent and physical motion, parallax, siderial time

Learning goals

- How can we orient ourselves in the cosmos? Name and explain the different coordinate systems (Horizon, Equatorial, Galactic)
- How can we change systems or how can we pass from a coordinate system to another? Transformations
- Do stars change position ? Yes, either apparent (refraction, aberration) or physical (proper motion, precession, nutation) effects.
- How can we measure distances in astronomy? Parallax
- How do I measure time in astronomy? Explain how time is measured via siderial time, apparent or mean time in regard to the Sun

In this chapter, we will speak about how we can orientate ourselves in the cosmos, which appears to not feature any fix points. In doing so we will dive deeper into different coordinate systems, distances and astronomical timescales.

1.1 Coordinate systems

1.1.1 Celestial Sphere

Celestial objects are placed on the 2D surface of an ideal sphere. The center of which coincides with Earth's center. They are thus visible from Earth by viewing the inside wall of the sphere. Circles measuring the circumference of this sphere are called *great circles*.

There are 2 kinds of perpendicular great circles:

- **Horizon** [N,S,W,E]
- **Meridian** [Zenith, Nadir, N, S]

With the *Zenith* being placed in perpendicular direction above the observer and the *Nadir* being placed in perpendicular direction below the observer. The circles as well as the observer can be seen in Figure 1.1.1.

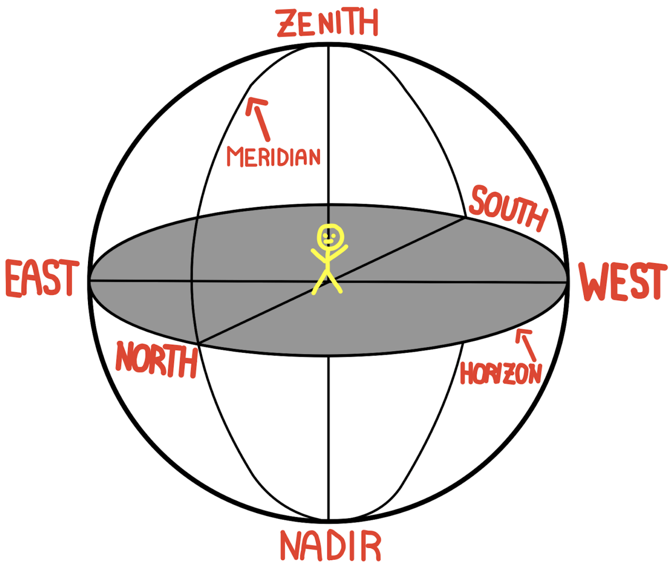


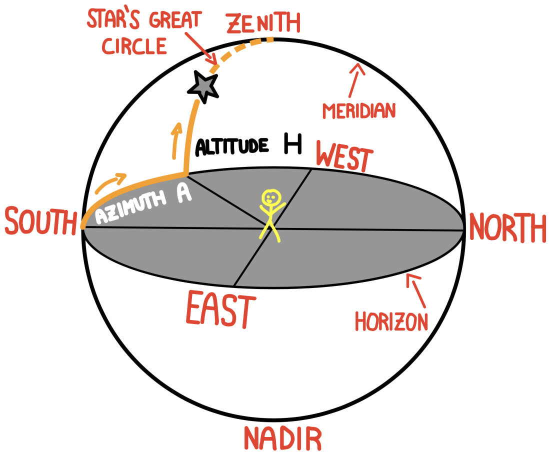
Figure 1.1.1: Celestial Sphere.

1.1.2 Horizon or Altitude/Azimuth system

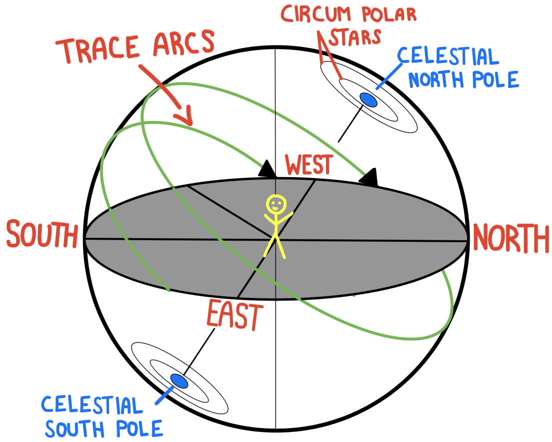
The most straight forward approach to determine the location of an object requires only two coordinates based on the observers local position and his horizon. The radial distance is not of interest in this system. See Figure 1.1.2a.

- **Altitude [H]:** The altitude H is defined as the angle measured from the horizon to the object along a great circle (not the meridian) passing through the Zenith. Therefore, z is the angle from the Zenith to the object and $H + z = 90°$.
- **Azimuth [A]:** The Azimuth is defined as the angle spanning from North eastwards/ South westwards until the great circles measuring the altitude intersects with the horizon.

The coordinates depend on the local observers latitude and longitude. It is therefore difficult to transform the coordinates to any other observer. Secondly, the stars are constantly moving, due to Earths rotation. Finally, the stars rise 4 minutes earlier every day, as a siderial Earth day (1 rotation around spin axis) only takes 23h56min. See Figure 1.1.3. The celestial north and south are equivalent to Earths north and south and thus intersect with the Earth rotational spin axis. When stars rise in the east they follow their so called trace arcs until they reach their highest point called *upper culmination (UC)*. Moving westwards and dipping behind Earth, they also reach an LC. This can be seen in Figure 1.1.2b.



(a) The two coordinates Azimuth and Altitude are displayed in regard to a reference star.



(b) Stars move along trace arcs (green). Here they reach a highest point above the horizon (UC).

Figure 1.1.2: Azimuth-Altitude system.

The Earth moves approximately 1° on its orbit around the Sun every 24h, meaning that Earth would have to rotate 361° to face the Sun again. Far distant stars experience a much lower effect resulting in them "rising" 4 minutes earlier relative to Earth. Therefore a siderial day is defined as the time between two UC.

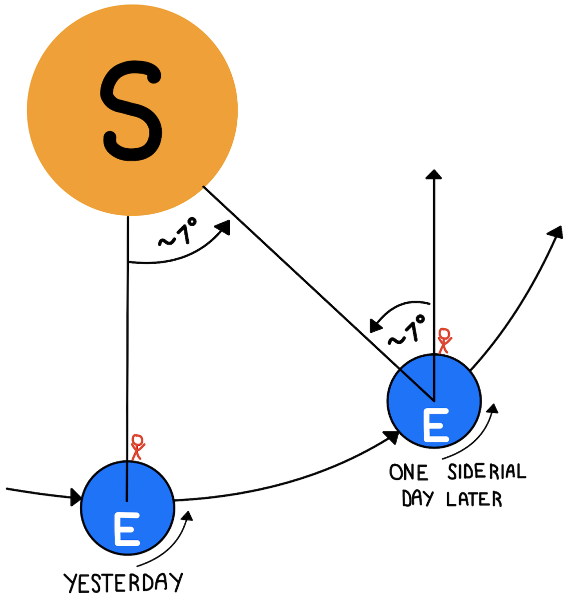


Figure 1.1.3: Siderial Day (360° turn) vs. Synodical Day (361° turn).

Important (Exam)

Please describe and discuss the Horizontal and the ECS coordinate systems.

1.1.3 Equatorial Coordinate System [ECS]

The ECS is based on the latitude - longitude system of Earth. Earth's local coordinates are given by longitude λ and latitude ϕ with $\lambda = 0$ for Greenwich and $\phi = 0$ for the equator. Again only two coordinates are necessary to describe a position at the sky.

- **celestial equator:** the projection of the Earth's equator onto the celestial sphere
- **hour circle:** the great circle passing through the object and the North Celestial Pole (NCP)
- **right ascension** [α]**:** α is an angular distance, measured eastward along the celestial equator from the Vernal equinox to its intersection with the hour circle. Is it measured in time: hours, minutes, seconds (HMS)
- **declination** [δ]**:** δ is an angular distance, measured positive to the north and negative to the south along the hour circle from the celestial equator. Is it measured as an angle: degrees, arcminutes, arcseconds (DMS)

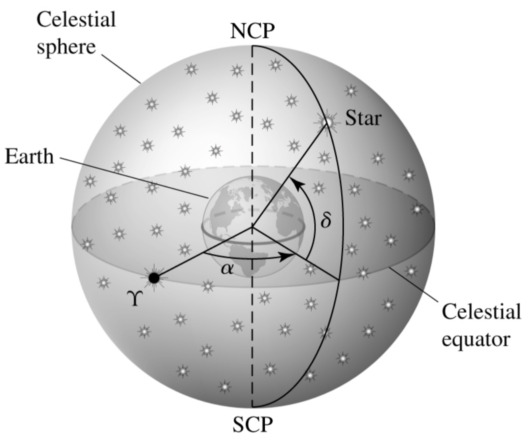


Figure 1.1.4: Equatorial Coordinate System, Caroll & Ostlie.

The conversion is as follows: **1° = 60' = 3600"** or **1° = $\frac{1}{15}$ hr = 4 min**
which by multiplying by 15 results in: **15° = 1 hr, 15' = 1 min, 15" = 1 sec**

So whereas the horizon system uses the horizon and a great circle passing through the Zenith, the ECS uses the actual projection of the north pole and equator onto the celestial sphere.

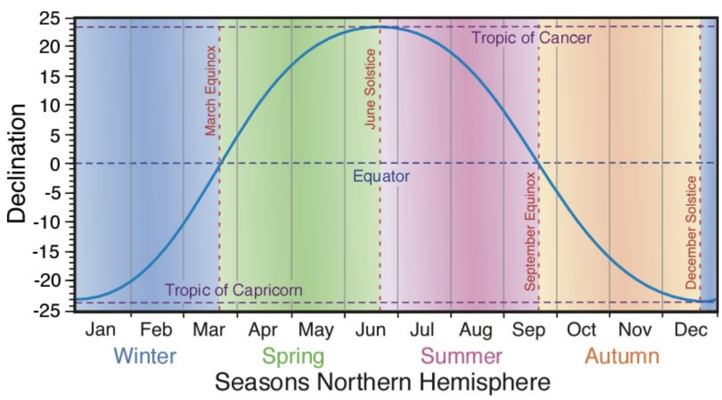


Figure 1.1.5: Seasons northern hemisphere, Prof. Santangelo slides.

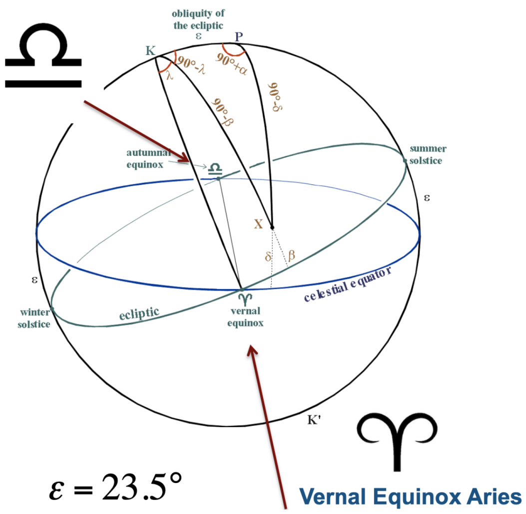


Figure 1.1.6: Equinox points and ecliptic, Prof. Santangelo slides.

The ecliptic describes the plane in which the Earth orbits around the Sun. The celestial sphere is inclined by 23°27' which causes Earth to have seasons. During orbit we reach the point closest to the Sun, called the *Perihelion*, at $1.47 \cdot 10^{11}$m and our most distant point, named the *Aphelion*, at $1.52 \cdot 10^{11}$m. Twice a year the celestial equator intersects with the ecliptic. These points are called *Vernal equinox* or *Autumnal equinox*. The most northern excursion of the Sun along the ecliptic is called "summer solstice", the southern most position has the name "winter solstice". As a result of the inclination of the ecliptic with respect to the celestial equator, the declination of the Sun changes during the year. At the June solstice, it reaches +23.5°, the most northern excursion, while at the December solstice it reaches -23.5°, the most southern position. Ultimately the Sun is visible for a longer period during summer.

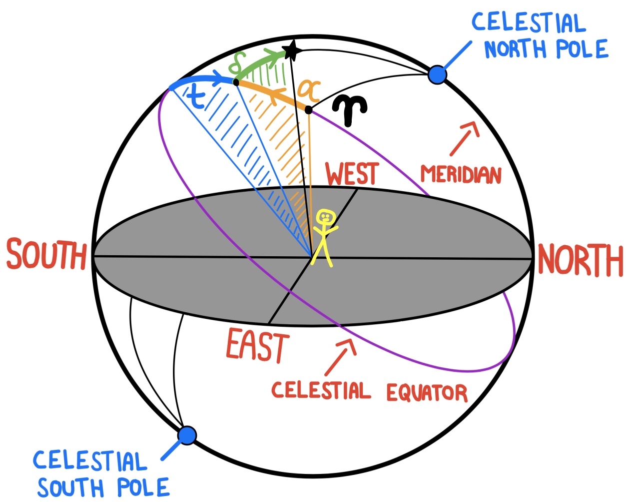


Figure 1.1.7: Siderial time and hour angle.

A further simplification uses the so called *hour angle [t]* which describes the angular separation westwards (in time) from the meridian to the hour circle used to fixate the star. The hour angle of the vernal equinox is called *sidereal time* θ. It is given by the right ascension (equinox to hour circle) and hour angle (meridian to hour circle) both of the object.

$$\theta = \alpha + t \tag{1.1.1}$$

1.1.4 Galactic coordinates

Again we only need two coordinates. This time the coordinate system is centered around the Sun. The quantities denoted with L and B are angular coordinates on the sphere, where B is the Galactic latitude while L is the Galactic longitude.

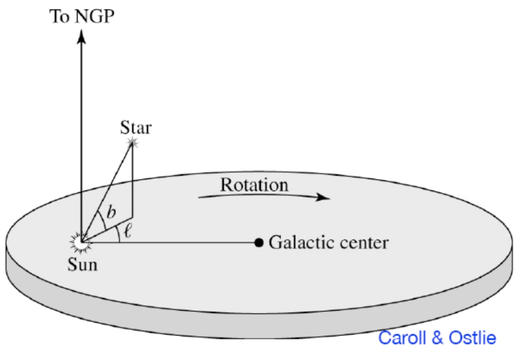


Figure 1.1.8: Galactic coordinates, Caroll & Ostlie

- **Galactic latitude [B]:** Counts positive towards the North up to 90° and negative in the southern direction
- **Galactic longitude [L]:** Counts from 0° to 360° for objects deviating from the line connecting the Sun and the galactic center

Stars along a fixed line of sight have the same L or B, which is why they can not be separated unless a third coordinate is introduced: the distance. The galactic center has the galactic coordinates [0°,0°].

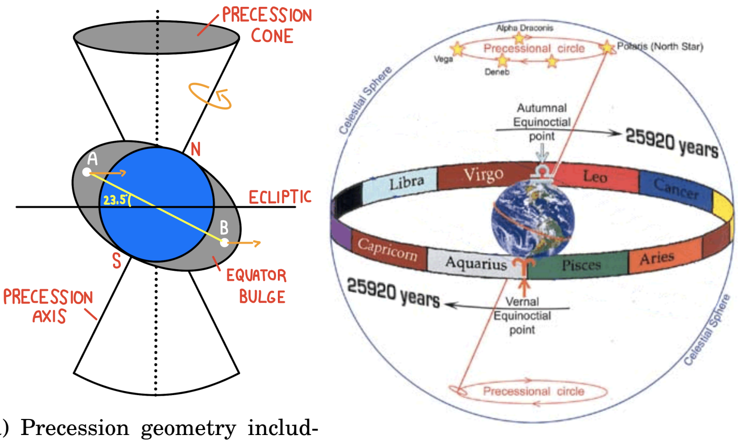
1.2 Variations and star positions

1.2.1 Rotation of the Earth

The rotation of the Earth is not fixed around one axis in space. Effects such as Precession and Nutation arise. Variations of the observed star positions may include effects such as Abberation, Refraction, Parallax.

Precession

Precession denotes the change in the direction of the rotation axis of an astronomical body e.g., the Earth. Tidal forces of bodies close to the ecliptic, i.e. Sun and Moon, create a torque $\dot{L} = M$ which enforces Earth's bulges towards the ecliptic plane. This causes the Earth to precess (rotation of spin axis around precession axis). Earth's precession period is 25770 years (Platonic year) while the NCP makes a slow circle around the precession axis. Ultimately, the coordinates, right ascension and the declination of a celestial body, will change. As a result, while we are closest to Polaris, the north star (within 1°) today, Vega will be the brightest star in the north in 12000 years time while Polaris will be 47° off axis. Further pertubation shifts include west and eastward shifts on the order off arcseconds/year. Thus, we consider coordinates taken at a certain point in time (reference) with the change of the coordinates added on top for a given time today.



(a) Precession geometry including cone and Earth's bulges.

(b) Precession calender.

Figure 1.2.1: Influence of Precession on Earth's night sky.

The epoch commonly used today for astronomical catalogs of celestial objects refers to the object's position at noon (UT) in Greenwich, (UK) on January 1st 2000. This reference date is called J2000 where J stands for Julian Calendar.

New coordinates:
1) $\Delta\alpha = M + N \cdot \in(\alpha)$
2) $\Delta\delta = N \cdot \cos(\alpha)$,

where M & N depend on T, T^2 and T^3 and are defined as:

$$M = 1^{\circ}.2812323T + 0^{\circ}.0003879T^2 + 0^{\circ}.0000101T^3 \tag{1.2.1}$$

$$N = 0^{\circ}.5567530T - 0^{\circ}.0001185T^2 - 0^{\circ}.0000116T^3 \tag{1.2.2}$$

with $T = \frac{(t-2000)}{100}$ and t = current date, specified in fractions of a year (e.g. 2021,8)

Nutation

Earth's precession nutates back and forth with period of 18.6 years due to the gravitational pertubation caused by the Moon. This is due to the fact that the Moon's orbit is inclined with respect to the ecliptic by 5.1°, resulting in the precession of the Moon's orbital plane. The amplitude of nutations is small: 7-9".

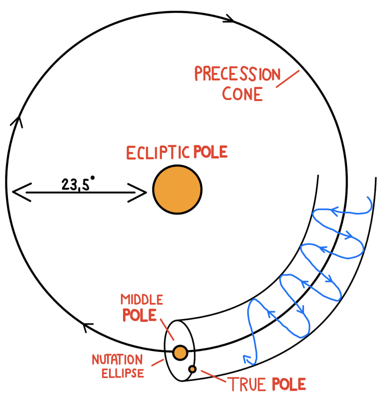


Figure 1.2.2: Influence of Nutation on Earth.

1.2.2 Variation of the observed star positions

Abberation

The observer sees light shifted in the direction of the observers direction of motion. The moving frame is not equal to the rest frame.

Refraction

Refraction means the change of direction of light resulting from the influence of the atmosphere along the line of sight. The atmospheric influence depends on its local value of humidity among other environmental parameters. Therefore the Sun for example can even be seen, when it has dipped below the horizon.

Proper motion

This variation means the change of the positioning of objects with respect to the static background. The motion can be broken down into a radial component, which can measured by Doppler shift in the spectrum and a tangential component. The tangential velocity, along the celestial sphere, appears as a slow angular change in its equatorial coordinates.

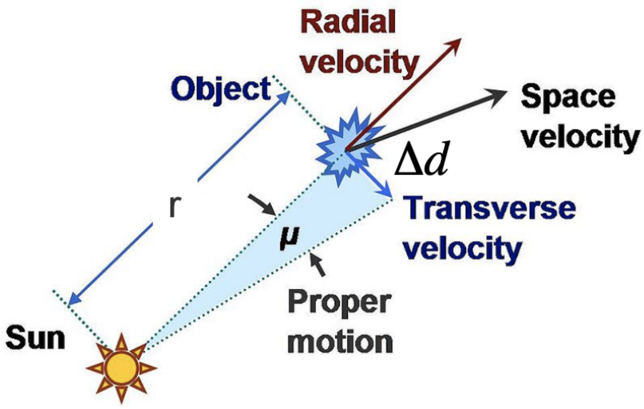


Figure 1.2.3: Proper motion of celestial objects.

$$\Delta d = v_\theta \cdot \Delta t \tag{1.2.3}$$

$$\Delta\theta = \frac{\Delta d}{r} \tag{1.2.4}$$

$$\mu = \frac{d\theta}{dt} = \frac{v_\theta}{r}. \tag{1.2.5}$$

1.2.3 Distances: Parallax

The motion of the Earth around the Sun generates the apparent motion of a star with respect to the very distant objects (e.g. Quasars = quasi stellar objects). The parallax is defined as the change of a star position due to the orbit of the Earth around the Sun. Depending on the position of the star, an ellipse, a circle (NCP) or a line (equator) will be traced in the celestial sphere during the year.

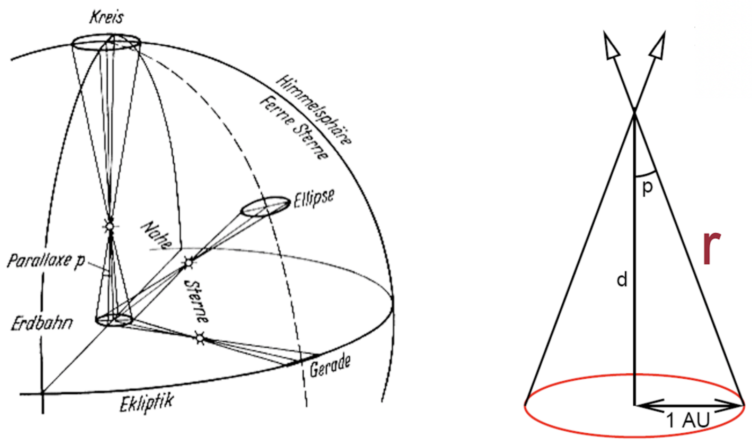


Figure 1.2.4: Distance parallax.

1 pc (parallax-second) corresponds to the distance of a star whose parallax is 1 arcsec for an angular radius of 1 AU. With **1 pc =** $3.086 \cdot 10^{16}$**m = 3,26 ly**. The closest star to to the Sun is Proxima Centauri at a distance of d = 1.3 pc.

1.3 Time

1.3.1 Apparent and mean sidereal time

The apparent sidereal time is determined by the position of the true vernal equinox. It is obtained by direct observation. When doing so, precession and nutation need to be taken into account as these incluence the position of the vernal equinox.
The *mean equinox* is determined by the position the vernal equinox would be if there were no precession or nutation. The *mean sidereal time* is then the hour angle of the mean equinox.

$$\theta_{\mathrm{a}} - \theta_{\mathrm{m}} = \Delta\Psi cos\epsilon \tag{1.3.1}$$

with ϵ being the obliquity of the ecliptic at the instant of observation and $\Delta\Psi$ being the nutation (in longitude).

1.3.2 Solar Time

In general it is more convenient to use the alternation of day and night: defining time according to the apparent motion of the Sun. The true (or apparent) solar time, T, based on the Sun's hour angle, does not flow at a constant rate. This has multiple reasons:

- The orbit of the Earth is not exactly circular, but an ellipse, which implies that the Earth's velocity along its orbit is not constant.
- The Sun moves along the ecliptic -not the equator- and therefore the right ascension does not increase at a constant rate: fastest at the end of December (4 min 27 s per day), slowest mid-September (3 min 35 s per day). A schematic illustration is given in Figure 1.3.1.

To compensate for this, a mean Sun, which moves along the celestial equator with constant angular velocity, is invented. It makes a full rotation in one year.

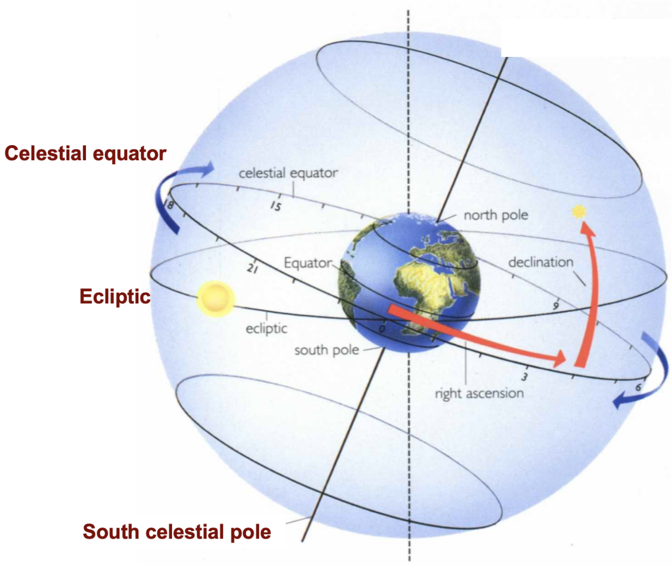


Figure 1.3.1: Illustration of Celestial Sphere including Ecliptic.

The mean solar time (T_M) is defined as the hour angle of the Sun + 12 hours. This 12 hour offset arises from making each day start at midnight. The hour angle or the mean sun is measured from the zenith (noon). The *Equation of time* (ET) is the defined as

$$ET = T - T_{\mathrm{M}} \tag{1.3.2}$$

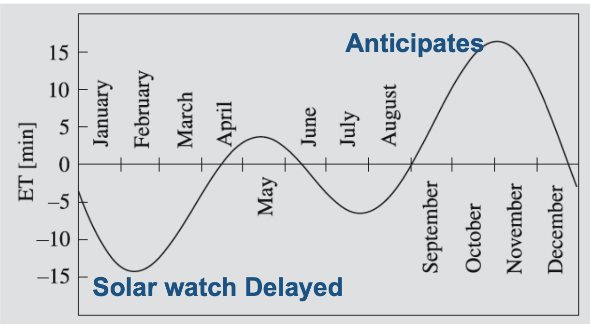


Figure 1.3.2: Visual display of Equation of time.

1.3.3 Time measurement: the year

Three exemplary cases are denoted to illustrated, in order to demonstrate the deviations of time measurements.

- **Sidereal year:** Time needed by the Sun to make one full revolution with respect to the background fixed stars. It corresponds to the true orbital period of the Earth equaling 365.25637 mean Sun days.
- **Tropical year:** Time needed by the Sun to rise at the vernal equinox (two consecutive times) equaling 365.24219 mean Sun days.
- **Gregorian year:** one year in the Gregorian calendar, equaling 365.2425 = 365 + 1/4 - 3/400 mean Sun days (Gregorian calendar, 1582)

1.4 Additional information

1.4.1 Transformations

The transformation from equatorial to horizontal coordinates is given by:

$$\textbf{Horizon}(A,z) \longrightarrow \textbf{Equatorial}(\delta,t)$$

$$\cos\delta\sin t = \sin z\sin A \tag{1.4.1}$$

$$\cos\delta\cos t = \cos z\cos\varphi + \sin z\sin\varphi\cos A \tag{1.4.2}$$

$$\sin\delta = \cos z\sin\varphi - \sin Z\cos\varphi\cos A \tag{1.4.3}$$

And conversely, the inverse transformation:

$$\textbf{Equatorial}(\delta,t) \longrightarrow \textbf{Horizon}(A,z)$$

$$\sin z\sin A = \cos\delta\sin t \tag{1.4.4}$$

$$-\sin z\cos A = \cos\varphi\sin\delta - \sin\varphi\cos\delta\cos t \tag{1.4.5}$$

$$\cos z = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos t \tag{1.4.6}$$

Where φ is the geographical latitude of the observation site

1.4.2 Astrometry

Satellite missions to do Astrometry (ESA):
1) **Hipparcos**: precision: 0,01", max. distance: 300 pc
2) **GAIA**: precission: 1 uas, distance: 10's of kpc = milky way (d = 30 kpc)

1.4.3 Fun facts

1) Mass of Black hole at center of Milky way: $4\cdot 10^6 M_\odot$
2) Cosmic microwave background fluctuations (CMB): $\frac{\Delta t}{t} = 10^{-5}$

Chapter 2

Celestial mechanics

Abstract. Introduction to the Kepler laws and how they are obtained. This also leads to the definition of orbits and multi-body systems.

Keywords: Kepler, mechanics, forces, geometry

Learning goals

- What are the Kepler laws? Learn the Kepler laws and how they can be derived from the conversation of energy and angular momentum.
- What type of orbits do I have in Astronomy? Learn the types of orbits depending on the mechanical energy of the system.
- How can we solve the multibody problem? What multibody problem means and a few simple systems.

The topic of the chapter is celestial mechanics, which deals with the motions of celestial objects, especially but not exclusively those of planets and stars. To describe the motion of these objects we will use Newton's and conservation laws. Yet, only the two-body problem could be solved analytically. More complex problems involving more bodies (N-body problems) are usually solved numerically and are applied in a wide variety of fields, for instance in the detection of exoplanets, galaxies dynamic and the dynamics of clusters of galaxies. It is also worth to mention that celestial mechanics is not only used to describe the motion of the planets, for example the known eight of the solar system, but also those of very eccentric objects like 2013 FT28 or other Extreme Kuiper Belt Objects that reach far beyond as what we would intuitively call our solar system.

2.1 Kepler's laws

One of the great achievement of human thinking has been the understanding of the motion of the planets around the Sun, which is described by the Kepler's laws. The three Kepler's laws, discovered by JOHANNES KEPLER (1571 – 1630) are as follows:

First law(1609)

A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse: f_1 and f_2 are the foci.

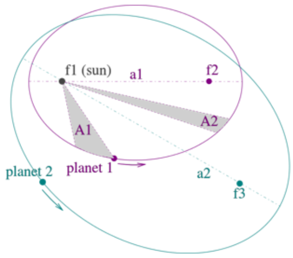


Figure 2.1.1: First and second Kepler's law

Second law(1609)

A line connecting a planet to the Sun ('Fahrstrahl') sweeps out equal areas A_1 and A_2 in equal time intervals (see figure 2.1). In other words: A line r connecting sun and planet sweeps out equal amounts of surface area in equal time:

$$A_1 \cdot \Delta t = A_2 \cdot \Delta t \tag{2.1.1}$$

Third law (1619)

The square of the orbital period P of a planet is proportional to the cube of the semi-major axis a of its orbit.

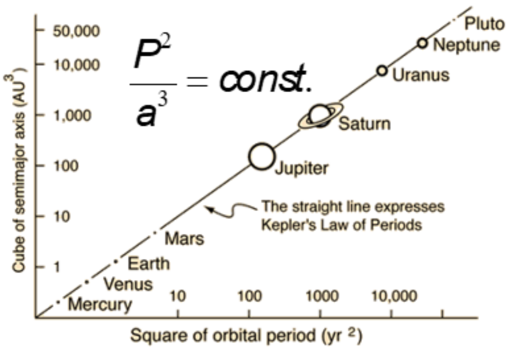


Figure 2.1.2: Third Kepler's law

And quantitatively:

$$\frac{P^2}{a^3} = \text{constant} \tag{2.1.2}$$

Important (Exam)

Know all three Kepler laws and apply them.

2.2 Two-body problem

2.2.1 Equation of motion

We will derive the equations of motion, where we will see that a two-body problem can effectively be reduced to a one-body problem. This is extremely handy and eases many calculations. Afterwards we will briefly discuss the results and learn about the types of orbits that are possible.

The force of gravity, introduced by SIR ISSAC NEWTON, is an attractive force between two massive bodies.

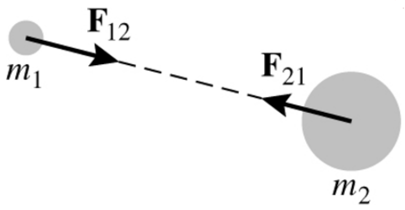


Figure 2.2.1: Newton's Law

Body 1 experiences a force from body 2 and vice versa (3rd law of Newton). Newton's law of gravity is quantitatively expressed as

$$\vec{F}_{21} = -Gm_1m_2\frac{\vec{r}_{12}}{\vec{r}^{\,3}_{12}} = -Gm_1m_2\frac{\hat{r}_{12}}{r^2_{12}} \tag{2.2.1}$$

$$\vec{F}_{12} = Gm_1m_2\frac{\vec{r}_{12}}{\vec{r}^{\,3}_{12}} = Gm_1m_2\frac{\hat{r}_{12}}{r^2_{12}} \tag{2.2.2}$$

where we used that $\vec{r}_{12} = \hat{r}_{12}r_{12}$ and defined $\hat{r}_{12}$ as the unit vector pointing from the center of body 1 to body 2 and r_{12} as the absolute distance between the two centers. If we consider our sun and Earth, we know that $M_\odot = 332.946 M_{Earth}$. The mass of the sun in SI units can be found in the appendix alongside some other useful and common values in astronomy.

Let us now consider a sun S and a planet P in a two-body system.

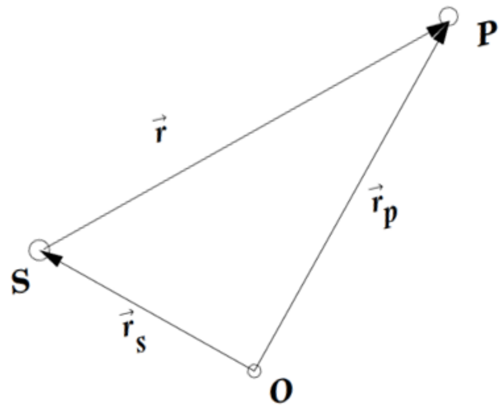


Figure 2.2.2: Sun and a planet as a two-body system

As seen in the figure above, O denotes the origin of the coordinate system, $\vec{r}_S(t)$ and $\vec{r}_P(t)$ the radial vectors of the sun and the planet respectively. Thus, the vector connecting both bodies can be obtained by$\vec{r}(t) = \vec{r}_P(t) - \vec{r}_S(t)$. The ultimate goal of this discussion is to find the solution of the two body problem, in other words: to find $r(t) = |\vec{r}(t)|$ by using Newton's law of universal gravitation. For that we go back to the expression of the forces (2.2.2) and write down the equations of motions like we always do by using Newton's law. We obtain for the sun:

$$M_S\ddot{\vec{r}}_S = M_S\ddot{\vec{a}}_S = \frac{GM_Sm_P(\vec{r}_P(t) - \vec{r}_S(t))}{|\vec{r}_P(t) - \vec{r}_S(t)|^3} = GM_Sm_P\frac{\vec{r}}{r^3} \tag{2.2.3}$$

$$m_P\ddot{\vec{r}}_P = m_P\ddot{\vec{a}}_P = -\frac{GM_Sm_P(\vec{r}_P(t) - \vec{r}_S(t))}{|\vec{r}_P(t) - \vec{r}_S(t)|^3} = -GM_Sm_P\frac{\vec{r}}{r^3} \tag{2.2.4}$$

We now apply a common strategy where we reduce a complex looking two-body problem to a one-body problem by introducing relative coordinates. We can obtain the equation for the relative motion simply by multiplying the second equation by M_S and the first equation by m_P and

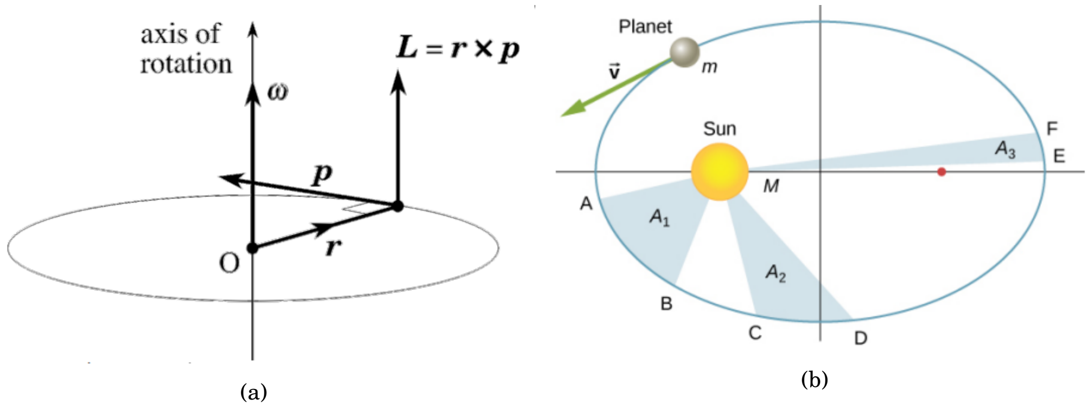


Figure 2.2.3: Momentum conservation

afterwards subtracting them from each other. When we now divide by $M_S + m_P$ we end up with

$$\mu \ddot{\vec{r}} = \mu \vec{a} = -GM\mu \frac{\vec{r}}{r^3} \tag{2.2.5}$$

where we introduced the following quantities

$$\mu = \frac{M_S m_p}{M_S + m_P} \tag{2.2.6}$$

$$M = M_S + m_P, \tag{2.2.7}$$

with M being the total mass of the system and μ the so called reduced mass. All in all we have:

Relative coordinates

The coordinates:

$$\vec{r}(t) = \vec{r}_1(t) - \vec{r}_2(t) \tag{2.2.8}$$

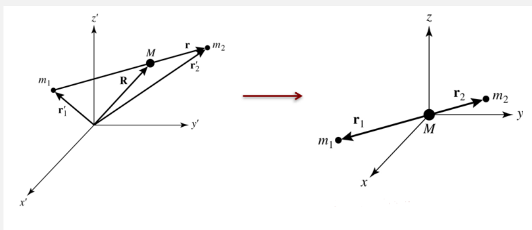
$$\vec{r}_1(t) = -\frac{m_2}{m_1 + m_2}\vec{r} \tag{2.2.9}$$

$$\vec{r}_2(t) = \frac{m_1}{m_1 + m_2}\vec{r} \tag{2.2.10}$$

and the masses:

$$\mu = \frac{M_S m_P}{M_S + m_P} \tag{2.2.11}$$

$$M = M_S + m_P \tag{2.2.12}$$



Now we can solve the problem comparably easily in the coordinate system in which the origin is in the center of mass since we have reduced it to a one dimensional problem. The planet moves as a body of reduced mass in the field of a central mass given by the sum of the masses. Again: the mass of the planet is much smaller than the mass of the sun.

Let us now consider the conservation laws. First, we begin with the conservation of energy

$$E = \frac{1}{2}\mu v^2 - GM\mu\frac{1}{r}, \tag{2.2.13}$$

consisting of the kinetic and potential term. The angular momentum is also conserved, i.e.

$$\vec{L} = \mu\vec{r} \times \vec{v} \tag{2.2.14}$$

where we set $\vec{v} = \dot{\vec{r}}$. Let us now introduce the specific angular momentum h,which is the angular momentum per unit of mass

$$\vec{h} = \frac{\vec{L}}{\mu} \tag{2.2.15}$$

Both the angular momentum $\vec{L}$ and the specific momentum $\vec{h}$ are orthogonal to $\vec{r}$ by definition, in other words: the motion is orthogonal to $\vec{L}$. We choose a coordinate system with the z-axis parallel to $\vec{L}$, i.e.

$$h = \frac{L_z}{\mu} \tag{2.2.16}$$

We will now express the equations of motion in terms of polar coordinates. They are defined as

$$\vec{r} = \begin{pmatrix} r\cos\phi \\ r\sin\phi \end{pmatrix} \tag{2.2.17}$$

or in terms of Cartesian coordinates

$$x = r\cos\phi \tag{2.2.18}$$

$$y = r\sin\phi \tag{2.2.19}$$

We will use r to denote the distance to the origin instead of ρ.

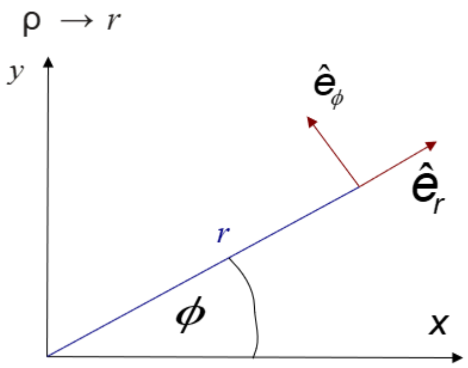


Figure 2.2.4: Polar coordinates and angular momentum

By using the unit vectors

$$\vec{e}_\phi = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}, \quad \vec{e}_r = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \tag{2.2.20}$$

we can then calculate the derivatives of (2.2.17) with respect to the time t by using the chain rule

$$\dot{\vec{r}} = \dot{r}(t)\vec{e}_r + r(t)\dot{\phi}(t)\vec{e}_\phi \tag{2.2.21}$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2)\vec{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\vec{e}_\phi \tag{2.2.22}$$

Therefore, the equations of motion become

$$m\ddot{\vec{r}} = m(\ddot{r} - r\dot{\phi}^2)\vec{e}_r + m(2\dot{r}\dot{\phi} + r\ddot{\phi})\vec{e}_\phi = -G\frac{mM}{r^2}\vec{e}_r \tag{2.2.23}$$

and the angular momentum in $\vec{e}_z$ becomes

$$\vec{L} = r\vec{e}_r \times m(\dot{r}\vec{e}_r + r\dot{\phi}\vec{e}_\phi) = mr^2\dot{\phi}\vec{e}_z = m\vec{h} \tag{2.2.24}$$

Thus, by dividing equation(2.2.23) by m and multiplying with $\vec{e}_r$, we obtain the radial equation of motion. Together with equation (2.2.24) we have

Equation of Motion(2D)

$$\ddot{r} - r\dot{\phi}^2 = -\frac{GM}{r^2} \tag{2.2.25}$$

$$r\dot{\phi} = h \tag{2.2.26}$$

The first one is the radial motion equation, the second is the azimutal motion equation. All in all, we have effectively reduced the complex, 3-dimensional equations (2.2.3) to a handy, 2-dimensional set of equations.

2.2.2 Discussion of Equation of Motion & conserved quantities

Here we will see that we can obtain the law of areas (2nd Kepler law). A line connecting a planet to the sun sweeps over equal areas in equal time intervals. This can be seen by considering the following geometry

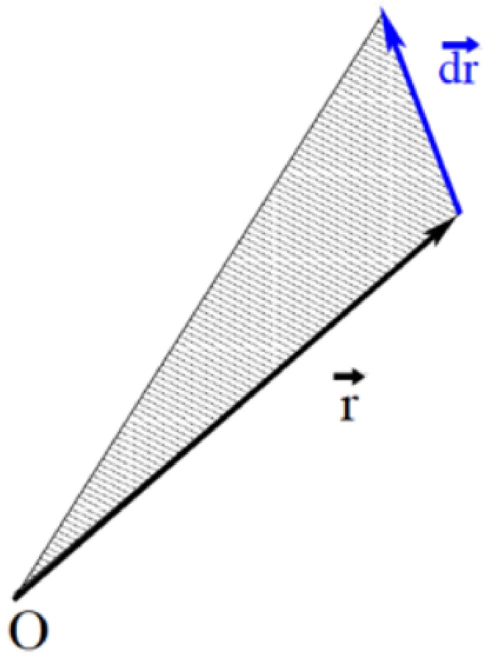


Figure 2.2.5: Geometry for derivation

The differential (very small) area that is swept is given by

$$dS = \frac{1}{2}|\vec{r} \times d\vec{r}|, \tag{2.2.27}$$

which is illustrated in the figure above: since $|\vec{r} \times d\vec{r}|$ would yield the total square and we are interested in the triangle, the factor of 1/2 is introduced. If we divide by dt, we get,

$$\dot{S} = \frac{1}{2}|\vec{r} \times d\vec{V}| \tag{2.2.28}$$

Using the definition of h in (2.2.15) and (2.2.16), this becomes

$$\dot{S} = \frac{1}{2}h \tag{2.2.29}$$

This is exactly the second law since the angular momentum is conserved; therefore, h is constant and the swept area is the same for equal time intervals. Note that the area is also independent of the mass of either body, which is quite remarkable.

2.2.3 Form of orbits

First, we will consider the energy conservation, which leads us to the effective potential and the so called radial equation of motion. From there, we can predict which forms the orbits can have. Let us now consider the conservation of energy. Gravity is a conservative force and therefore the total mechanical energy E of the orbiting body is conserved.

$$E = E_{\text{p.g}} + E_{\text{kin}} = \text{constant} \tag{2.2.30}$$

where $E_{\text{p.g}}$ is the gravitational potential energy of mass m in the field of M, i.e.

$$E_{\text{p.g}} = -G\frac{mM}{r}. \tag{2.2.31}$$

On the other hand, the kinetic energy is given by

$$E_{\text{kin}} = \frac{1}{2}m|\dot{\vec{r}}|^2 = \frac{m}{2}(\dot{r}^2 + r^2\dot{\phi}^2) = \frac{m}{2}\dot{r}^2 + \frac{L^2}{2mr^2}. \tag{2.2.32}$$

The kinetic energy is divided in two parts. One originates from the movement along the radius and the other one is associated with the tangential, or azimuthal motion. This part depends on r but neither on the angle ϕ, nor on the velocity v.

$$E_{\text{kin}}^{\text{radial}} = \frac{m}{2}\dot{r}^2 \tag{2.2.33}$$

$$E_{\text{kin}}^{\text{tangential}} = \frac{L^2}{2mr^2} \tag{2.2.34}$$

Since it depends only on r, the tangential component can be included in the potential energy obtaining an effective potential:

$$V_{\text{eff}} = E_{\text{p}}^{\text{Eff}} = -G\frac{Mm}{r} + \frac{L^2}{2mr^2} \tag{2.2.35}$$

Let us now introduce the conservation of angular momentum in the radial motion equation. We introduce the effective potential which includes the angular momentum expressed in terms of h from (2.2.16)

$$V_{\text{eff}}(r) = -\frac{K}{r} + \frac{h^2}{2r^2} \tag{2.2.36}$$

where $K = GM$.

Important(Exam)

Write down and explain the effective potential, explain all the terms and draw the plot as function of the mechanical energy $\epsilon = \frac{E}{m}$

Now we want to rewrite the equations of motion(2.2.25)into a equation for the effective potential. For that we express (2.2.25) also in terms of h and get

$$\ddot{r} + \frac{K}{r^2} - \frac{h^2}{r^3} = 0 \tag{2.2.37}$$

If we multiply this by the velocity v and integrate over the time dt, latter equation reads

$$\int \ddot{r}\dot{r} + \int \frac{K}{r^2}\dot{r}dt - \int \frac{h^2}{r^3}\dot{r}dt = 0 \tag{2.2.38}$$

$$\Leftrightarrow \int \ddot{r}dr + \int \frac{K}{r^2}dr - \int \frac{h^2}{r^3}dr = 0 \tag{2.2.39}$$

$$\Leftrightarrow \int \ddot{r}dr + -\frac{K}{r} + \frac{h^2}{2r^2} = 0 \tag{2.2.40}$$

$$\Leftrightarrow \frac{1}{2}\dot{r}^2 + \int \ddot{r}dr + -\frac{K}{r} + \frac{h^2}{2r^2} = \frac{1}{2}\dot{r}^2 \tag{2.2.41}$$

When we now use that $F = m\ddot{r}$m,identify the potential energy as $E_{pot} = -\int F dr$ and introduce the quantity of the specific energy of a planet as $\epsilon = \frac{E}{m}$, then we obtain

$$\frac{1}{2}\dot{r}^2 - \frac{K}{r} + \frac{h^2}{2r^2} = \epsilon. \tag{2.2.42}$$

By using the expression of the effective potential, we obtain

$$\dot{r}^2 + 2V_{\text{eff}}(r) = 2\epsilon \tag{2.2.43}$$

This is the *radial equation*. It turns out that the effective potential contains a lot of information about the shape of an orbit. As seen in the figure below, for small distances r, the repulsive centrifugal force dominates.

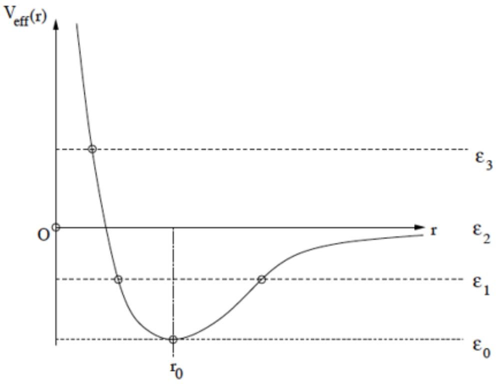


Figure 2.2.6: Effective Potential

For the value $V_{\text{eff}} = \epsilon_0$, the planet orbits the star in a *bound* and *circular* orbit. When, $V_{\text{eff}} = \epsilon_1$, , then the orbit is also *bound*, but in a *elliptical*. When reaching ϵ_2, the trajectory is *marginally* bound and describes a parabola, while for ϵ_3 it is *unbound* hyperbola. In summary:

V_{eff}	(Un)bound	Shape
ϵ_0	Bound	Circular
ϵ_1	Bound	Elliptical
ϵ_2	marginally bound	parabolic
ϵ_3	unbound	hyperbolic

Table 2.1: Orbits

Let us take a closer look onto these orbits. When writing $\dot{r}$ in a rather unconventional way

$$\dot{r} = \frac{dr}{d\phi}\frac{d\phi}{dt} \tag{2.2.44}$$

we can rewrite equation(2.2.43) to

$$\dot{r}^2 - \frac{2K}{r} + \frac{h^2}{r^2} = 2\epsilon \tag{2.2.45}$$

$$\Leftrightarrow \left(\frac{dr}{d\phi}\frac{d\phi}{dt}\right)^2 \frac{1}{h^2} - \frac{2K}{h^2 r} + \frac{1}{r^2} = \frac{2\epsilon}{h^2} \tag{2.2.46}$$

By defining $u = \frac{1}{r}$, we see from the chain rule that

$$\frac{dr}{d\phi} = -r^2 \frac{d}{d\phi}\frac{1}{r} = -\frac{1}{u^2}\frac{du}{d\phi} \tag{2.2.47}$$

Also from equation (2.2.24), we see

$$h = \frac{1}{u^2}\frac{d\phi}{dt} \tag{2.2.48}$$

Let us now plug the previous two equations (2.2.47) and (2.2.48) into (2.2.46) to obtain the result

$$\left(\frac{du}{d\phi}\right)^2 + u^2 - 2\frac{Ku}{h^2} = \frac{2\epsilon}{h^2}. \tag{2.2.49}$$

By derivating with respect to ϕ again, this becomes due to chain rule

$$\frac{d^2u}{d\phi^2} + u = \frac{K}{h^2} \tag{2.2.50}$$

This has exactly the form of an inhomogeneous harmonic oscillator. One family of solutions is

$$r = \frac{P}{1 + e\cos(\phi - \phi_0)}, \tag{2.2.51}$$

which is the equation of an ellipse with p denoting the is the semi-latus rectum and e denoting the eccentricity of $0 \leq e \leq 1$. For $e > 0$,this corresponds to the case of $V_{\text{eff}} = \epsilon_1$ from Table 1. The case $e = 0$ yields circular orbits. Rearranging previous equations shows

$$p = \frac{h^2}{K} = a(1 - e^2) \tag{2.2.52}$$

, which is exactly Kepler's first law, i.e. all planets move on ellipses. Another solutions of (2.2.50) is

$$r = \frac{2d}{1 + cos(\phi)}, \tag{2.2.53}$$

which describes a parabola where d is the distance of the closest approach to the parabola's focus. The third possible solution is given by,

$$r = \frac{a(e^2 - 1)}{1 + e\cos(\phi)}, \tag{2.2.54}$$

a hyperbola. It is particularly striking that we obtained all predicted cases from earlier just by solving the radial equation analytically. What is also quite interesting is that all solutions are conic sections.

Conic sections are the borders of a plane that intersects a symmetric double cone. In figure 2.2.6, the first image shows a parabola, the second one an elliptical (or even circular) intersection and the third illustration depicts a hyperbola.

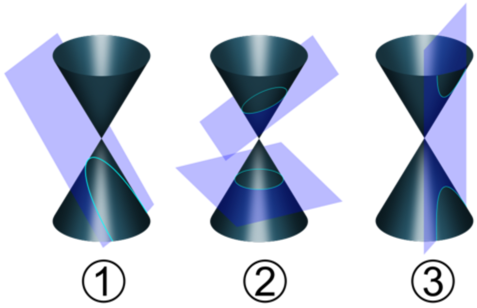


Figure 2.2.7: Conic Sections

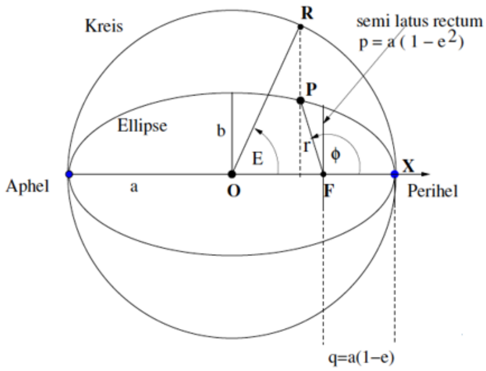


Figure 2.2.8: Geometry

2.2.4 Geometry

Let us take a moment to understand the geometry of the Kepler problem with which we are dealing with.

In the figure 2.2.7 , we can see that F is the focus point of the ellipse, i.e. where the sun is located. P is the planet that moves around the sun; r denotes the distance between both bodies, a is the semimajor axis while b stands for the semiminor axis, thus $a \geq b \geq 0$, where the equality holds in the case of a circle. Another characteristic quantity is the eccentricity which we already encountered in the previous chapter. It is defined by

$$e = \left(1 = \frac{b^2}{a^2}\right)^{\frac{1}{2}}. \tag{2.2.55}$$

The perihelion $q = a(1-e)$ is the closest point to the sun, the aphelion $Q = a(1+e)$ on the other hand is the most distant point to it. The "True Anomaly" ϕ denotes the angle between the perihelion and the planet and the "Eccentric Anomaly" $E^{'}$ stands for the angle between the perihelion and the planet from the center of the ellipse. Note that we named the eccentric anomaly $E^{'}$ here to avoid confusion with the energy E.

The energy and the angular momentum in this geometry should naturally lead us back to at least one of Kepler's laws. Since at the perihelion r_p and aphelion r_a the velocity is orthogonal to the position vector: $\vec{V} \perp \vec{r}$, the conservation of angular momentum and the conservation of energy yield

$$L = \mu\vec{r} \times \vec{p} = \mu r_p V_p = \mu r_a V_a \tag{2.2.56}$$

$$E = \frac{1}{2}\mu V_p^2 - \frac{K}{r_p} = \frac{1}{2}\mu V_a^2 - \frac{K}{r_a} \tag{2.2.57}$$

where the subscripts denote the quantities at the perihelion or aphelion. Therefore, get by using $r_p = a(1-e)$ and $r_a = a(1+e)$, we get:

$$L^2 = \mu^2 r_p r_a V_a V_p = \mu^2 a^2 (1-e^2) V_a V_p. \tag{2.2.58}$$

For bound orbits, there is a balance between gravitational force and centrifugal force, i.e. for ellipses the velocities must fulfill[1]

$$V_p = \sqrt{\frac{GM}{a}}\sqrt{\frac{1+e}{1-e}} \tag{2.2.59}$$

$$V_a = \sqrt{\frac{GM}{a}}\sqrt{\frac{1-e}{1+e}} \tag{2.2.60}$$

Combining these with equations (2.2.57) and (2.2.58), we obtain the important relations

$$L = \mu\sqrt{GMa(1-e^2)} \tag{2.2.61}$$

and

$$E = -\frac{GM\mu}{2a}. \tag{2.2.62}$$

Integrating the expression for the area of the ellipse from equation (2.2.29) leads us to

$$\dot{S} = \frac{1}{2}h \tag{2.2.63}$$

$$\Leftrightarrow \int_0^P \dot{S}dt = \int_0^P \frac{h}{2}dt \tag{2.2.64}$$

$$\Leftrightarrow \pi ab = \frac{h}{2}P, \tag{2.2.65}$$

where we used that the area of am ellipse is given by $S = \pi ab$. Together with the fundamental relation for ellipses

$$b = a\sqrt{1-e^2}, \tag{2.2.66}$$

we can combine the relations (2.2.61), (2.2.65) and (2.2.66) to obtain Kepler's third law

$$P^2 = \frac{4\pi^2 a^3}{GM} \tag{2.2.67}$$

that may be more familiar in the form that we saw earlier:

$$\frac{P^2}{a^3} = \text{constant} \tag{2.2.68}$$

The velocities to reach certain orbits, for instance when a test objects starts from the earth, can then easily be calculated by Kepler's third law. For a circular orbit, the velocity

$$P^2 = \frac{4\pi^2 R_E^3}{Gm_E} \Leftrightarrow v_1 = \sqrt{\frac{Gm_e}{R_E}} \tag{2.2.69}$$

where R_E is the radius of the earth. For a parabolic velocity, the gravitational potential on earth's surface must be equal to the kinetic energy that is needed to escape, i.e.

$$\frac{1}{2}v_2^2 = \frac{Gm_E}{R_E} \Longrightarrow v_2 = 2\sqrt{\frac{Gm_E}{R_E}} = \sqrt{2}v_1. \tag{2.2.70}$$

[1]For a derivation of these velocities see e.g. https://www.sternfreundemuenster.de/pdf/peraph497.pdf

This can be explained by the fact that parabolic orbits are the energetic lowest states of unbound trajectories (see Table 1). In other words, it is the speed needed to "break free" from the gravitational attraction of the earth.At any position the orbiting body has the "escape velocity" for that position. It is remarkable that the escape velocity, or parabolic velocity is irrespective of which direction the test particle is moving in. For an elliptical orbit, the test particle needs a velocity v_3 anywhere in

$$v_1 < v_3 < v_2, \tag{2.2.71}$$

while for a hyperbola it has to satisfy

$$v > v_2. \tag{2.2.72}$$

2.2.5 Kepler Equation

Let us go back to the radial equation (2.2.42), reading

$$\frac{1}{2}\dot{r}^2 - \frac{K}{r} + \frac{h^2}{2r^2} = \epsilon \tag{2.2.73}$$

which we can rewrite be defining $A = 2\epsilon$, $B = K$, $C = -h^2$ as

$$\dot{r}^2 = A = \frac{2B}{r} + \frac{C}{r^2}. \tag{2.2.74}$$

Let us now separate the variables dr and dt and integrate

$$\int_{r_0}^{r} \frac{dr^{'}}{\sqrt{A + \frac{2B}{r^{'}} \frac{C}{r^{'2}}}} = \int_{t_0}^{t} dt^{'} \tag{2.2.75}$$

We substitute,

$$r = a(1 - e\cos E^{'}), \tag{2.2.76}$$

where $E^{'}$ is again the eccentric anomaly (see previous chapter) and also

$$a = -\frac{B}{A} \tag{2.2.77}$$

$$e = \left(1 - \frac{AC}{B}\right)^{\frac{1}{2}} \tag{2.2.78}$$

Carrying out the integral (2.2.75) with these substitutions, which we will not do here since it is not very insightful, and choosing that at $t = t_0$ the planet is at the perihelion $r_0 = r_p$, yields the Kepler equation:

Kepler Equation

$$M = \frac{2\pi}{P}(t - t_0) = E^{'} - e\sin E^{'} \tag{2.2.79}$$

where M is the mean anomaly, $E^{'}$ is the eccentric anomaly, P is the period and e is the eccentricity.

For $E^{'} = 0$ the planet is at the perihelion and for $E^{'} = \pi$ at the aphelion respectively. Kepler's equation embodies the geometric properties of the orbit of a body subject to a central force. It is a transcendental equation. We cannot solve it for $E^{'}$ algebraically. Numerical analysis and series expansions are required to evaluate $E^{'}$. It can be shown that

$$\tan\left(\frac{\phi}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E^{'}}{2}\right), \tag{2.2.80}$$

while the distance follows equation (2.2.51).

2.2.6 Orbital Elements

We are closing this subsection with some remarks on orbital elements. These are just a set of parameters that are required to uniquely identify a specific orbitwhich follows Kepler's laws. One uses the following quantities: 2 orbit elements a, e (semimajor axis, eccentricity), 3 angles (they fix the orientation of the orbits) and 1 time origin, set it for example at the time where the planet passes the perihelion. With these six parameters, we are able to describe the planet's position uniquely.

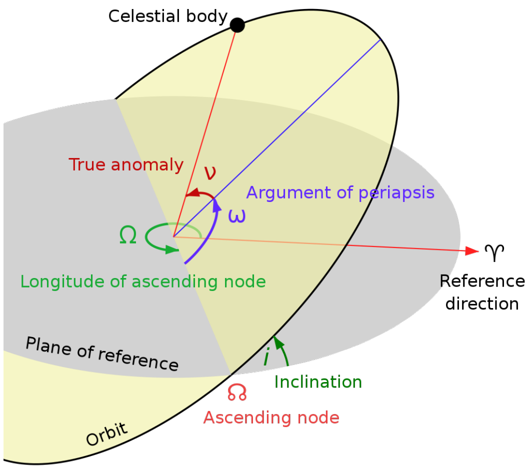


Figure 2.2.9: Orbital elements

Let us briefly mention every quantity: two elements describe the shape of the orbit (ellipse), three define the spatial orientation of the orbit and one is the time:

1. Eccentricity e: Shape of the ellipse - how much does it vary from a circle?
2. Semimajor axis a: the sum of the periapsis and apoapsis distances divided by two (see figure 2.2.7).
3. Inclination i: vertical tilt of the ellipse with respect to the reference plane
4. Argument of periapsis ω: Defines where the low point, perigee, of the orbit is with respect to the surface of the central body.
5. True Anomaly ν (earlier we called it ϕ): defines the position of the orbiting body along the ellipse at a specific time
6. Time T_p: The point in time where the planet passes the perihelion

With these six parameters we can uniquely define the location $\vec{r}$ and its velocity $\vec{v}$. On the other hand, by observing these quantities, we can determine the trajectory. Observing these quantities is often rather difficult because an observer on earth is only able to measure two angles which define the position of the celestial body on the hemisphere in the night sky. In fact, there are certain methods to compensate this by measuring the position several times, which led to a huge success in 1801 where astronomers could relocate Ceres using this method. The current orbital parameters of all elements in the solar system can be found at on website of NASA (link).

2.3 N-body problem

After we have treated the two-body problem in the previous chapter, we will now move on to a more complex problem involving more objects. In the simple case we saw that the equations of motion for the sun and the planet given by (2.2.3) and (2.2.4). After introducing relative coordinates, we ended up with

$$\mu\ddot{\vec{r}} = \mu\vec{a} = -GM\mu\frac{\vec{r}}{r^3}, \tag{2.3.1}$$

where $\mu = \frac{M_S m_p}{M_S + m_p}$ is again the reduced mass and $M = M_S + m_p$ the total mass. For, $N > 2$, the N-body problem does not have an exact analytical solution. Only particular cases have been solved, for example three masses on a equilateral triangle, three masses on a line rotating on a common center of mass etc. (link). As an example for such a more complex system, we look at the solar system with its eight planets. For each planet i, we need to take into account the gravitational attraction of the sun and of the other planets with masses m_i and positions $\vec{r}_i$:

$$\ddot{\vec{r}}_i = -GM_S\frac{\vec{r}_i}{|\vec{r}_i|^3} + \frac{\vec{F}_i}{m_i} \tag{2.3.2}$$

$$\vec{F}_i = -\sum_{j=1, i\neq j}^{8} GM\frac{\vec{r}_{i,j}}{|\vec{r}_{i,j}|^3} \tag{2.3.3}$$

where $i \in [1,8]$ and $\vec{r}_{i,j} = \vec{r}_i - \vec{r}_j$. . In addition, we need to take into account the equation of motion of the sun. As we said these equations can only be solved numerically, which is obvious just by bearing the number of terms in mind that appear in each equation. The approach that is usually chosen is the one that uses perturbations, meaning that the main solution is the ellipse which is perturbed by the other planets. For example, the eccentricity of the earth's orbit is influenced by the force of the other planets. Today we end up with a result of $e = 0.0167$.

Let us not briefly examine on a three body problem. A key insight here are the equipotential lines. These are lines on which the gravitational potential is constant. We look at a special case of this scenario where the equipotential lines of a three masses that satisfy the condition

$$m_3 << m_1, m_2. \tag{2.3.4}$$

A third mass much smaller than the other two will feel the force

$$\vec{F}_3 = -Gm_3\left(m_1\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|^3} + m_2\frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|^3}\right) \tag{2.3.5}$$

If the effect of the third mass is negligible, the other two masses will follow Keplerian orbits around the center of mass, with a certain period and frequency. In the most simple cases these will be circles. We can then choose a rotating reference coordinate system centered around the center of mass and rotating with frequency ω. In this system, the position vectors of the two main masses are invariant and m_3 moves in the so called Roche potential

$$\phi_{\text{eff}} = -G\left(m_1\frac{1}{|\vec{r}_3 - \vec{r}_1|^3} + m_2\frac{1}{|\vec{r}_3 - \vec{r}_2|^3}\right) - \frac{1}{2}\omega s^2 \tag{2.3.6}$$

where the term $\frac{1}{2}\omega s^2$ comes from centrifugal forces as we have chosen a rotating coordinate system. Then s is the distance of the third mass from the rotation axis. Equipotential lines of the effective potential which are generated by two masses as seen in the the rotating coordinate system centered in the centre of mass are illustrated in the following figure:

Here, CM denotes the center of mass. What is interesting is that there are five points, the Lagrange points L1, L2, L3, L4 and L5, where there is local equilibrium. L1, L2, L3 are unstable, while L4 and L5 yield stable positions. They are especially useful for telescopes as they remain in a stable position relative to the sun and earth over a long period of time. The James Webb Telscope (JWT) will be set up in one of the Lagrangian points. When looking closely at Figure 2.12, one can see two closed lines that look like droplets. These are the Roche lobes of both masses. If both are stars for example,every mass that leaves the Roche lobe of one star, for example the less massive one, it will be accreted by the more massive star and will be transferred to it through the Lagrangian point L1 between them. In the case of a neutron star and a massive star this leads

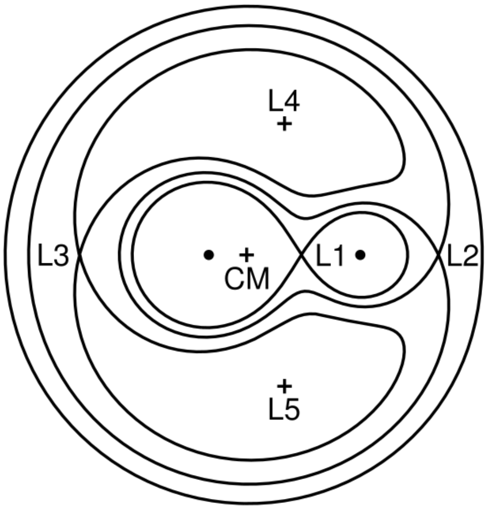


Figure 2.3.1: Roche potential in a rotating coordinate system

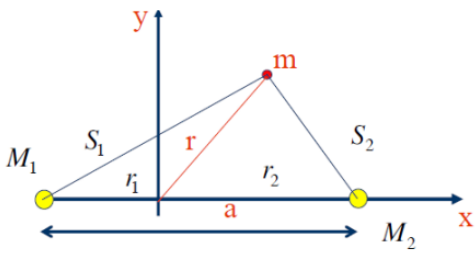


Figure 2.3.2: Gravity in a close binary system

to a spin up of the neutron star and it becomes what we call a pulsar. We will not look at this overflow quantitatively. First we start by considering gravity in a close binary system. We define the variables as shown in Figure 2.13. Let us choose a reference system which is corotating with the center of mass at the origin, also called the barycenter.

From Keplerian mechanics we know that

$$M_1 r_1 = M_2 r_2 \tag{2.3.7}$$

$$r_1 + r_2 = a, \tag{2.3.8}$$

with all variables like in figure 2.3.2 If we consider the two stars as point masses of infinitesimal size in the x-y-plane we obtain the angular velocity

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} \tag{2.3.9}$$

If we look at the corotating frame with

$$F_{centrifugal} = F_{grav} \tag{2.3.10}$$

$$\Leftrightarrow mr\omega^2 = G\frac{Mm}{r^2} \tag{2.3.11}$$

$$\Leftrightarrow \omega = \sqrt{\frac{G(M_1+M_2)}{r^3}}, \tag{2.3.12}$$

we can combine equation 2.3.9 and equation 2.3.12,

$$\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \sqrt{\frac{G(M_1+M_2)}{a^3}} \tag{2.3.13}$$

if we assume a stable corotation of system m at a distance $r = a$ away from the barycenter. Instead of forces we can use the potentials ϕ as

$$F = -\nabla\phi \tag{2.3.14}$$

holds in general. Hence by defining the centrifugal potential ϕ_c and the gravitational potential ϕ_g

$$\phi_c = -\frac{1}{2}m(\vec{\omega}\times\vec{r})^2 \tag{2.3.15}$$

$$\phi_g = -G\left(\frac{M_1 m}{S_1} + \frac{M_2 m}{S_2}\right) \tag{2.3.16}$$

we receive as the effective potential, also known as the Roche potential

$$\phi(\vec{r}) = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_1|} - \frac{1}{2}m(\vec{\omega}\times\vec{r})^2 \tag{2.3.17}$$

This is the effective potential experienced by a particle having mass m at a distance $r = |\vec{r}|$ away from the barycenter of the binary system. If we want to find the stable points, we look for curves with

$$\phi(\vec{r}) = \text{constant} \tag{2.3.18}$$

This can again be seen in figure 2.3.1 The Lagrangian points L1, L2, L3, L4 and L5 indicate positions where all gravitational forces nullify each other: a free force body will stay at this position forever.

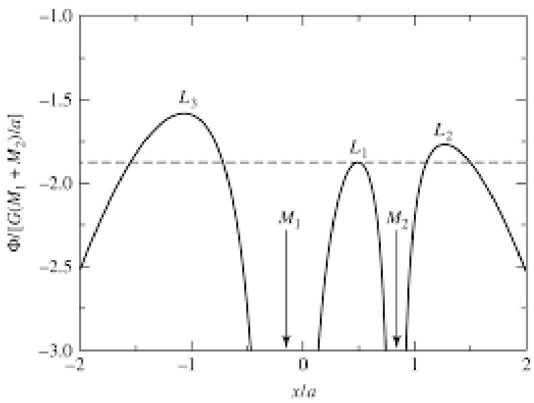


Figure 2.3.3: Potential lines of ϕ are shown as solid lines

The particle bound to one of the stars if its kinetic energy E satisfies

$$E < \phi_{\text{eff}}(L1). \tag{2.3.19}$$

On the other hand, it is not bound to stars but to the binary itself when

$$\phi_{\text{eff}}(L2) > E > \phi_{\text{eff}}(L1), \tag{2.3.20}$$

loosely speaking it is just bound to the center of mass of M_1 and M_2. Note that gas when spilling over will dissipate energy by compression. The gas looses energy and goes on a bound orbit around the new central star.

Chapter 3

Astronomical Instruments

Abstract. In this chapter, we introduce the techniques and instruments of astronomical observations. In particular, we will focus on Radio, Optical and X-ray telescopes. We will also introduce instruments and observatories of astroparticle physics.

Keywords: light, electromagnetism, optical, telescopes

Learning goals

- What are the fundamental quantities of light that we measure?
- What is the electromagnetic spectrum? Describe the energies and wavelengths of different ranges.
- What are the basics of optical light detection?
- What is angular resolution, what is energy resolution?
- How do optical telescopes work, and which are the current and future optical telescopes?
- How do X-ray telescopes work, and which are the current and future X-ray telescopes?

In general, light features two different characteristics. It can be either be seen as a wave or as a particle. As particles, photons are messengers which give an insight into the universe. Other messengers are cosmic rays (charged particles), neutrinos or even gravitational waves.
The energy of light can be expressed with the following relations

$$\lambda = \frac{c}{\nu} \quad , \quad E = h\nu, \tag{3.0.1}$$

where λ is the wavelength, c is the speed of light, ν is the frequency and h is Planck's constant. The spectrum of the sun for example is close to that of a black body at $T = 5780\,\mathrm{K}$. This corresponds to a peak intensity of 1.5 eV. The human eye is most sensitive at 2-3 eV, which is likely a result of evolution. The spectrum generally ranges from 10^{-5} eV to 10^{5} eV, however we have already discovered particles with energies as extreme as 10^{19} eV or even higher. Table 3.1 shows the ranges and properties of the electromagnetic spectrum, whereas table 3.2 shows the SI prefixes that are commonly used in high energy astrophysics.

Region	λ(Å)	λ (cm)	Frequency (Hz)	E (eV)
Radio	$> 10^9$	> 10	$< 3 \times 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \times 10^9 - 3 \times 10^{12}$	$10^{-5} - 0.01$
IR	$10^6 - 7000$	0.01-7×10^{-5}	$3 \times 10^{12} - 4.3 \times 10^{14}$	0.01-1.5
visible	7000-4000	$7 \times 10^{-5} - 4 \times 10^{-5}$	$4.3 \times 10^{14} - 7.5 \times 10^{14}$	1.5-3
UV	4000-10	$4 \times 10^{-5} - 10^{-7}$	$7.5 \times 10^{14} - 3 \times 10^{17}$	$3 - 10^3$
X-Rays	10-0.1	$10^{-7} - 10^{-9}$	$3 \times 10^{17} - 3 \times 10^{19}$	$10^3 - 10^5$
Gamma Rays	<0.1	$< 10{-}9$	$>3 \times 10^{19}$	$> 10^5$

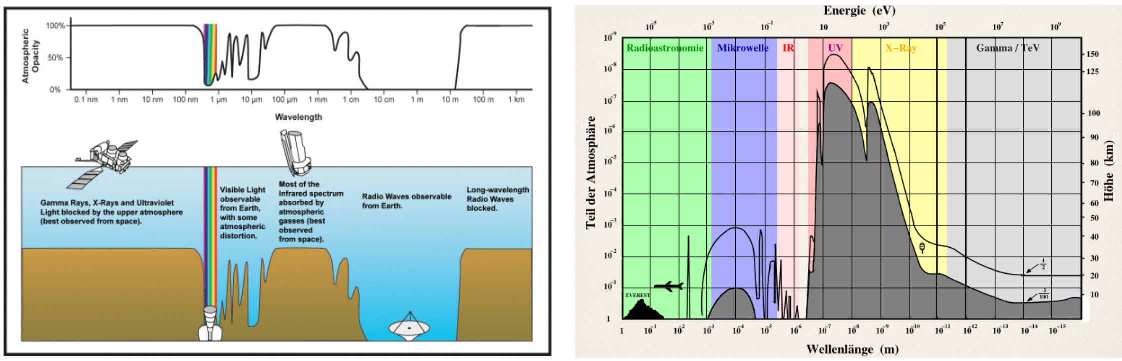
Table 3.1: Ranges and characteristics of the electromagnetic spectrum.

Prefix name	Prefix symbol	Energy
kilo	k	keV $\widehat{=}$ 10^3 eV
mega	M	MeV $\widehat{=}$ 10^6 eV
giga	G	GeV $\widehat{=}$ 10^9 eV
tera	T	TeV $\widehat{=}$ 10^{12} eV
peta	P	PeV $\widehat{=}$ 10^{15} eV
exa	E	EeV $\widehat{=}$ 10^{18} eV
zetta	Z	ZeV $\widehat{=}$ 10^{21} eV
yotta	Y	YeV $\widehat{=}$ 10^{24} eV

Table 3.2: SI prefixes commonly used for photon energies.

3.1 Observation on Earth

Radiation is absorbed in Earth's atmosphere depending on its frequency and on the altitude, as illustrated in the sketch in Figure 3.1.1a. Radio [10^5 Hz, 10^{11} Hz] and optical [400 THz, 800 THz] frequencies pass the atmosphere almost unhindered, whereas anything in-between or with shorter wavelength is absorbed almost completely. This is why generally any observation tool that works in the range above the optical one is space based. The only exception is extremely high energy radiation as these particles create showers of photons and electrons which rain down on to earth and can thus be detected.



(a) Atmospheric windows for the electromagnetic wavelengths on ground.

(b) Transmissivity of electromagnetic waves through the atmosphere.

Figure 3.1.1: Electromagnetic waves and the atmosphere.

3.1.1 Role of Earth's atmosphere

The high-energy gamma-rays, X-rays, and UV light do not pass through the upper atmosphere since they are absorbed. Visible light can be observed with some distortion from the atmosphere, whereas most of the infrared light is absorbed by water vapor and other gases present in the atmosphere. Radio waves are observable from the earth, as they are not disturbed by atmospheric gases but long-wavelength radio waves are blocked by the charged particles in the ionosphere. Figure 3.1.1b visualises the transparency of the atmosphere as a function of wavelength.

3.2 Optical Telescopes

In this chapter we introduce the techniques and instruments of astronomical observations. In particular, we will focus on radio, optical and X-ray telescopes. We will also introduce instruments and observatories to detect astroparticle physics. We distinguish two types of telescopes: the refractor and the reflector telescopes.

a) Refractor telescopes

A **refractor** (lens telescope) uses a collective lens. In physics, refraction is the change in direction of a wave passing from one medium to another. For light, refraction follows Snell's law, which states that, for a given pair of media, the ratio of the sines of the angle of incidence α and angle of refraction β is equal to the ratio of phase velocities $\frac{v_1}{v_2}$ in the two media, or equivalently, to the indices of refraction $\frac{n_1}{n_2}$ of the two media.

$$n_1 \cdot \sin(\alpha) = n_2 \cdot \sin(\beta) \quad , \quad n = \frac{c}{v_\lambda} \tag{3.2.1}$$

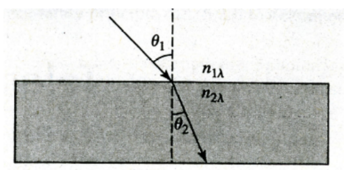


Figure 3.2.1: Refraction of light through a thick slab.

To simplify the following considerations, we assume parallel rays. This assumption is justified, as we think of the source to be very far away.

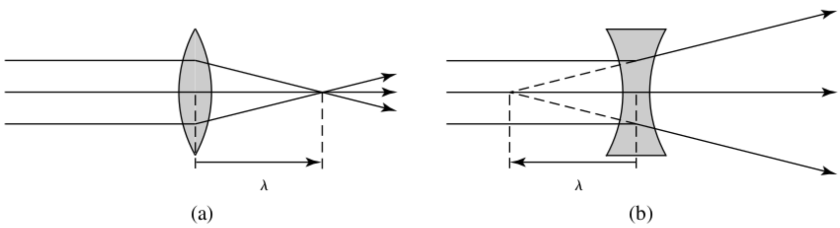


Figure 3.2.2: (a) Parallel rays converging through a convex lens. (b) Parallel rays diverging through a concave lens.

Now we can use the **lensmaker's equation**

$$\frac{1}{f} = (n_\lambda) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \tag{3.2.2}$$

$$\frac{1}{f} = \frac{1}{g} - \frac{1}{b} \tag{3.2.3}$$

with R_1 and R_2 being the curvature radii of the lens.

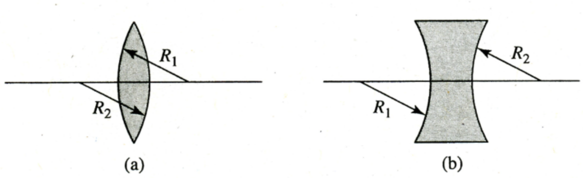


Figure 3.2.3: Two radii of focus of (a) convex lens (b) concave lens.

Both concave (diverge) and convex (converge) lenses have 2 foci. In the case of $\theta_1 = \theta_2$ we talk of reflection. Here, f denotes the focal length, which is independent of the wavelength λ. The quantity F stands for the position of the focal plane.

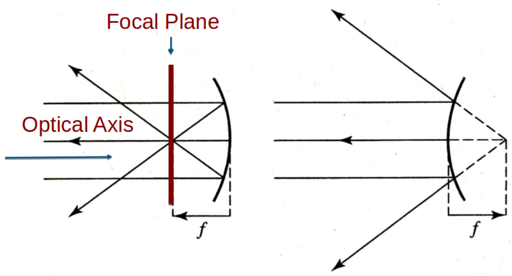


Figure 3.2.4: Focal plane of mirrors.

Sometimes two sources are very close together, which makes it hard to separate them. The sources are then called *diffuse* (as opposed to point sources). The two sources then differ by the impinging angle θ. The difference of focus along the focal plane is then given by

$$y = f \cdot \tan(\theta) \approx f \cdot \theta \tag{3.2.4}$$

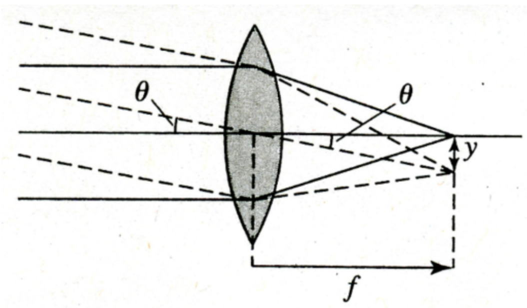


Figure 3.2.5: The plate scale relation.

This is also explains why refractors are so long, as y is proportional to f. The angular separation is

$$\frac{d\theta}{dy} = \frac{1}{f} \tag{3.2.5}$$

and allows for the minimum angular resolution to be reconstructed. Two well known space based telescopes are:

- ROSAT (DLR): Achieves a minimum angular resolution of 5"
- CHANDRA (NASA): Achieves a minimum angular resolution of 0.5"

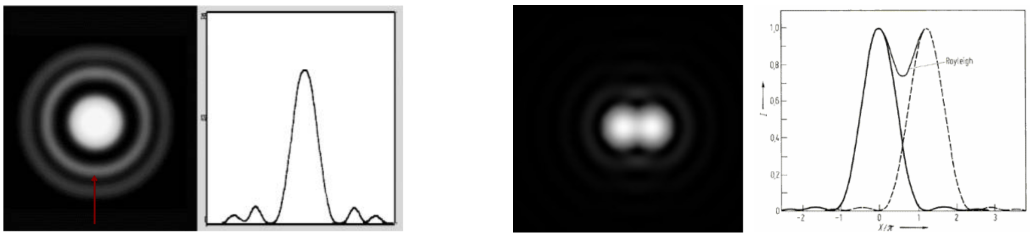
The smaller the aperture diameter (D), the less light can be collected by lens making it harder to see or resolve the images. The limiting factor is the diffraction at the lens edge. This results in many secondary maxima or so-called **airy discs**:

$$\sin \alpha = m \frac{\lambda}{D} \tag{3.2.6}$$

where m is the value for the according maxima or minima, see Table 3.3.

State and order	value m
Minima 1	1.22
Maxima 1	1.635

Table 3.3: First two entries of order values for maxima or minima when calculating the diffraction angle.



(a) Single source.

(b) Two sources which are just resolved.

Figure 3.2.6: Diffraction patterns of (a) a single source and (b) two sources which are just resolved.

From this relation, the criterion for being able to distinguish between two sources can be defined. The **Rayleigh criterion** states that: The first maxima of the second source needs to lie within in the first minima of the first source. Following equation 3.2.6, a radio telescope for example has worse resolution than an optical telescope of the same size because $\lambda_{\text{radio}} > \lambda_{\text{optical}}$.

$$\tan \alpha_{\text{min}} = 1.22 \frac{\lambda}{D}. \tag{3.2.7}$$

Important (Exam)

Calculate the minimal angle or distance that two sources in the sky need to have in order to be resolved.

Instrument	D	λ	α_{min}
Eyes	6 mm	5500 Å	0.4’
Small Telescope	13.8 cm	5500 Å	1.0”
Hubble ST	2.5 m	4000 Å	0.04”
Keck Hawaii	10 m	4000 Å	0.01”
Effelsberg Radio Telescope	100 m	21 cm	9’

Table 3.4: Angular resolution of different instruments.

A radio telescope will have resolution in the order of arcmin, whereas the optical achieves a resolution in the magnitude of arcsec. The limit achievable is constrained by the atmosphere down to a few arcsec. To help out with this, we use adaptive optics. When using **interferometry** radio telescopes, one can achieve values as low as 0.001" since D in 3.2.7 is very large. This does not work for X-rays though, as it is hard to collect particles with such small wavelengths with the needed high precision. Furthermore, the limit from earlier was a theoretical one, but practically it is much harder to collect X-rays (systemic effects). The flux (here J) is proportional to the surface of the aperture. This again is also proportional to the inverse of the focal length. With the focal ratio of f over D, J is given by

$$J \propto \left(\frac{D}{F}\right)^2. \tag{3.2.8}$$

The typical angular resolution of some telescopes and energy ranges is shown in Table 3.5.

Telescope	Angular resolution
Radio	0.2°
Array Radio telescope	0.1”- 0.001”
Microwaves (WMAP)	0.2°- 0.3°
X-ray (Chandra)	0.5”
Hard X-ray	12”
GeV (Fermi)	0.2°- 3°
Hess (TeV)	0.1°

Table 3.5: Angular resolution in different wavelength bands.

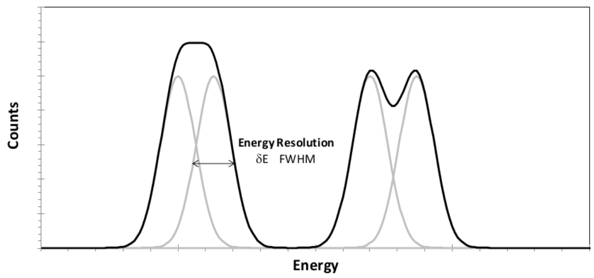


Figure 3.2.7: Energy resolution and FWHM of a pulse height distribution.

As an example, XMM-NEWTON has worse angular resolution than CHANDRA, however the illumination (flux) is much better. It is therefore able to gather spectra from much fainter stars and therefore to provide a much better energy distribution (SED). This is measured by

$$\frac{E}{\Delta E} \tag{3.2.9}$$

where ΔE is the full-width-half-maximum (FWHM), which is a known quantity from the introductory courses. The medium energy resolution is anything from 50 to 500, whereas high is anything above 1000. The magnification by a refractor geometry is given by

$$V = \frac{f_{\text{lens1}}}{f_{\text{lens2}}} = \frac{\alpha_{\text{lens1}}}{\alpha_{\text{lens2}}} \tag{3.2.10}$$

There is an important lens error that limits the resolution, called the **Chromatic aberration**. Different focal lengths are produced by different wavelengths. This is especially problematic for refractors as their resolution depends on λ. In this case, a so-called **achromat** can fix this error up to a certain threshold. The achromat concentrates all wavelengths back to one focus point. Another, more general problem with refractors is, that due to their size, the lenses as well as the whole construction is extremely heavy and difficult to manufacture.

3.2.1 Illumination

The amount of light energy per second focused onto a unit area of the resolved image is called Illumination J, which is proportional to the surface of the aperture. It describes the light-gathering power of the telescope.

$$J \propto \frac{\pi D^2}{4}$$

The dimension of the image is proportional to f^2, therefore the illumination must be inversely proportional to f^2

$$J \propto \frac{1}{f^2}$$

Combining above equations, we arrive at the following:

$$J \propto \frac{1}{F^2} = \frac{D^2}{f^2} \tag{3.2.11}$$

where F is the focal ratio.

b) Reflector telescopes

Contrary to the previously discussed setup, a **reflector** consists of a multi-mirror construction. In general, the primary mirror will be a paraboloid (concave) which concentrates the light on to a secondary mirror to extend the focal length. The main layouts are shown in Figure 3.2.8, they are

- **Newton:** The second mirror is plane parallel
- **Cassegrain:** Secondary is hyperboloid; longer focal length, compact construction
- **Riittchey–Chréttiien:** both hyperboloid. Today's standard for large telescopes
- **Schmidt-Cassegrain:** Aspherical collector lens. Today's amateur telescope.

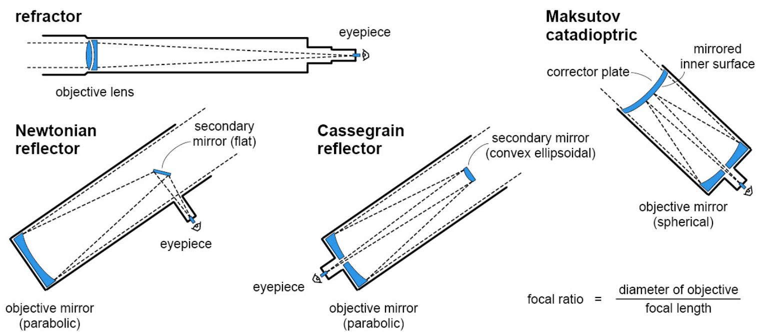


Figure 3.2.8: Geometries of different telescope types.

Important (Exam)

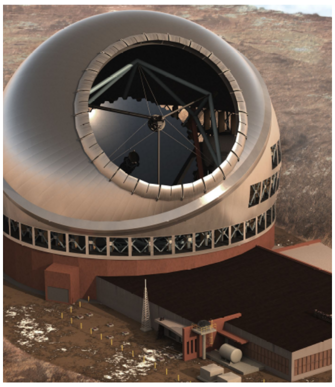
Describe different telescope set-ups (Newton, Cassegrain, Schmidt-Cassegrain, etc.).

3.3 Telescope mounting

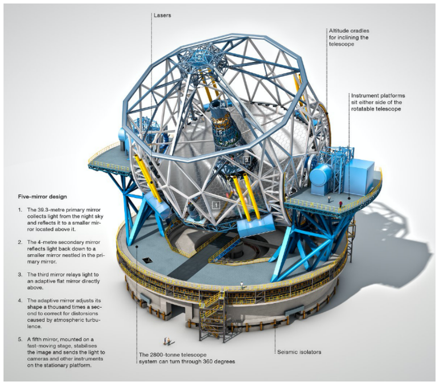
It is necessary for the telescope to be pointed at a fixed region of the sky for an extended period of time so that it can collect enough photons to produce a high-resolution, deep-sky image of faint objects. There are two basic types of telescope mounts.

- **Altitude-azimuth mount** An alt-az mount moves in two axes, the vertical (altitude) and horizontal (azimuth) to the horizon. One of the advantages is the mechanical simplicity of the mount design and light construction. This type of mounting is mostly used in modern telescopes, driven by a computer.
- **Equatorial mount** The equatorial (parallactic) mount accounts for the Earth's rotation. It has a polar axis that is aligned to the north celestial pole (NCP), which allows it to swing in an east-west direction. A second axis perpendicular to this allows the telescope to swing in a north-south direction. An advantage of this type of mounting is that the motion of the stars has to be compensated around the polar axis only.

Here there are problems with mechanical stability due to weight and temperature dependency. Solutions are thick mirrors, adaptive optics and active optics (change of mirror shape in real time with actuators). This is also achieved by mosaic mirror (many little mirrors), see e.g. the future 30m telescope (USA) or the ELT (ESA) in Figure 3.3.1.



(a) The Thirty Meter Telescope



(b) ELT - Extremely Large Telescope.

Figure 3.3.1: Examples of modern telescopes.

With that being said, we want to close the discussion on types of telescopes and want to briefly explain two different concepts dealing with corrections. One concept is the *adaptive optics*, which we have used this notion several times already.

Lense errors and spectrographs

The atmosphere causes problems such as seeing where the planar wavefront interacts with the particles in the atmosphere due to turbulences. Therefore, the light path is changed (and with that the position of star changes respectively). There are two methods to correct for this effect:

- **speckle interferometry:** Applied after the data is collected, it is part of the ata analysis. In order to correct for the blur, the image is taken in extremely short time steps (0.001 s.). This way the pictures can be processed afterwards and the original source is filtered out. But the short time leads to the obstacle that only very bright sources can be monitored, as faint sources will not emit enough flux for the telescope to capture during such period.

- **adaptive optics:** This is the correction of wavefront by changing the surface of the mirror in order to compensate the wavefront distortion. Corrections are made every 0.01 s. The feedback is given to a computer who performs real time calculations and applies adjustments.

The second method however requires a bright star with known coordinates to calibrate the system. With no star in the apparent window one can create a laser guide star. The light is reflected from the atmosphere, so that the computer can calculate the corrections.

Last, but not least, we mention the spectrograph. A spectral decomposition can be performed with a prisma or grating. For a prisma the refraction index is antiproportional to λ^2. Diffraction gratings are often used as dispersive elements, transmission-gratings or more often as reflection-gratings. These setups achieve interference by

$$d(\sin(\alpha) - \sin(\beta)) = m\lambda \tag{3.3.1}$$

where d is the grating index and m is the order. An advantage of reflective gratings is the concentration of light in higher orders of m and higher efficiency for larger angular dispersion. The resolution of these is given by

$$R = \frac{\Delta\lambda}{\lambda} = m \cdot N \tag{3.3.2}$$

Echelle-gratings are groove shaped and are optimized for the use at high incidence angles and therefore in high diffraction orders. A few known telescopes using spectrographs are:

- HUBBLE SPACE TELESCOPE (HST)
- VLTI

3.4 X-ray astronomy

As we have previously seen, electromagnetic radiation with high energy does not pass the atmosphere. An example of high energy radiation is X-ray.
The first satellite mission took place in 1962 with a Geiger-Müller counter. The flux count was much higher than expected. It exceeded the optical flux by a factor of about 100. The first X-ray source was SCO-X1 (Scorpio, X-ray first source). While doing so, a diffuse X-ray background was also measured.
A major problem with X-ray astronomy is the reflection of X-rays, which is zero (refractive index ≈ 1). This means that normal telescope lenses can not be used. The solution is given by *collimators*. These channel/constrain photons by entering into a certain tunnel. Basically, these tunnels focus the rays. Therefore, the detector can be smaller while having a higher intensity as more particles are being collected. This results in the detection of fainter objects and less background noise. It turns out that the X-rays can be reflected if they impinge on the mirrors with tiny grazing angles. These angles are given by

$$\theta \propto \frac{\sqrt{\rho}}{E} \tag{3.4.1}$$

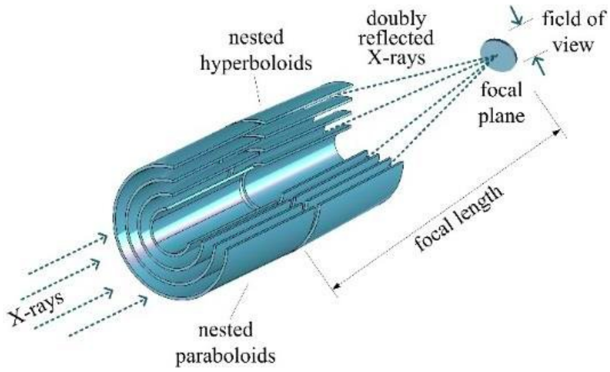


Figure 3.4.1: Wolter-I telescope configuration.

and are of the order of 1° to 2°. This setup is called a *Wolters telescope*. Although there are 3 types, the W1 configuration is of main interest here. Reading off from the above relation for the angle, higher energy requires smaller angles. This is a problem since we can not choose f as big as we want due to the impossibility to launch arbitrarily large rockets. Hence, we need to concentrate on ρ or equivalently n, which is defined as as

$$n = \sqrt{1-\rho} = \sqrt{1 - \frac{nZr_e}{\pi} \cdot \lambda}, \tag{3.4.2}$$

that depends on the atomic number Z. This is why dense material such as Gold or Iridium are commonly used. In the W1 setup, the particles are reflected twice. The first mirror is usually a paraboloid mirror while the second one is hyperboloid. The aperture of the mirror is therefore given by

$$A = 2\pi R \cdot l \cdot \sin\theta \tag{3.4.3}$$

with l = length of paraboloid mirror. The geometric collective area is then

$$A = \pi(R^2_{\text{outer}} - R^2_{\text{inner}}). \tag{3.4.4}$$

Important (Exam)

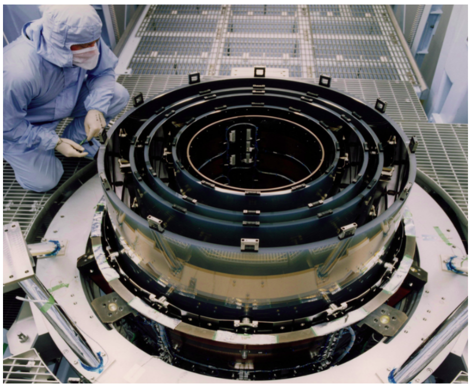
Explain the basic principles of X-ray telescope and describe W1 set-up.

To increase the effective area, the mirrors can be nested, i.e. many shells can be aligned coaxially. CHANDRA for example has 4 of such shells that can be seen in Figure 3.4.2a. It is a very stable system with thick and heavy mirrors, that is why using more than 4 shells is difficult.

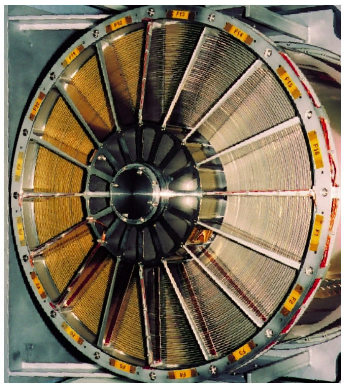
Note that 4 "perfect" (thick) mirrors results in a high resolution. XMM-NEWTON on the other hand has 58 shells (see Figure 3.4.2b), but they are thin mirrors and therefore not as stable. Also, achieving perfect alignment is more difficult as the weight of the mirrors bends them slightly. Thus, the spatial resolution is worse but the collecting area is larger, so spectral analysis of very faint objects is possible. This can be seen as a trade off and a rule of thumb is: *High resolution and low collecting area or visa versa.*

Telescope	High spatial resolution for imaging	Large effective area	Spectral analysis of faint sources
CHANDRA	yes	no	no
XMM-NEWTON	no	yes	yes

Table 3.6: Trade off between imaging and spectral analysis capabilities at the example of the CHANDRA and XMM-NEWTON space telescopes.



(a) The 4 nested thick mirrors of the CHANDRA telescope.



(b) The 58 nested mirrors of the XMM-NEWTON telescope.

Figure 3.4.2: Examples of telescopes using the Wolter type I configuration.

Future satellites like ATHENA are expected to combine the best of both worlds. The follow up from ROSAT, namely EROSITA, also features multi mirror optics (7 telescopes, each with 54 shells) and is designed to perform spectral analysis. The resolution is 16", which is worse than the one of CHANDRA. However, it has the best sensitivity ever and is supposed to map the whole sky finding billions of sources.

Before closing X-ray instruments and moving to the radio spectrum, we look at CCDs (Charge Coupled Devices). This principle is based on the photo effect in silicon crystals. Electrons are collected by electrodes marking the counts. This allows for high quantum efficiency. Illumination results in a cloud of electrons and the charge is collectively moved through the system by falling potential as the electrons migrate towards the anode. In general, incident radiation energy of $\approx 1\,\text{eV}$ equals to 1 electron-positron pair, which means that X-rays at $1\,\text{keV}$ give 1000 pairs.

When observing bright sources, signals are still being shifted while new signals already arrive. This results in lines in the final image. The EROSITA resolution has 384x384 pixels for 1° of the sky. Future techniques include silicon pore optics or DepFET which allows for every photon to create a signal and shortens the read out time. It is going to be faster than CCDs.

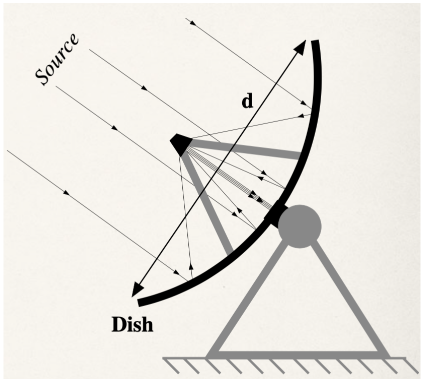
3.5 Radio astronomy

Learning goals

- What are the different energy spectral ranges? Which of them can be measured from the ground, which from space?
- How does radio interferometry work?
- Which are major radio telescopes?
- What about telescopes at other wavelengths?
- How do ground based Cherenkov telescopes and TeV photon detection work?

The second range in the electromagnetic spectrum that we want to discuss is the radio spectrum. Single antenna telescopes such as the Radio telescope in Effelsberg or Arecibo in Puerto Rico (crashed in Dec. 2020) or RATAN 600 in Russia are restricted to the Rayleigh criterion for resolution, see Equation 3.2.7.

(a) The Effelsberg Radio telescope.



(b) Parabolic antenna surface for max. signal gain.

Figure 3.5.1: Radio antenna set-up.

Naturally, a radio telescope will have a huge dish (on the order of 100m diameter), however the observed wavelength (on the order of cm) is also much larger than previously investigated. This in turn results in a resolution of multiple arcminutes. In order to achieve arcsecond resolution, the dish would have to feature a diameter greater than 50 km. This can obviously not be manufactured. Parabolic antenna surfaces are commonly used, as they allow to amplify the incoming signal. To do so, the roughness of the antenna surface must be of the order of the observed wavelength or shorter to gain good reflection. This is known as the antenna gain.

$$G = 10 \log \left(\frac{2d}{\lambda} \right)^2 \tag{3.5.1}$$

3.5.1 Radio interferometry

A parallel wave front might impinge on an array of telescopes. If this happens at an angle θ, then the two telescopes (array diameter D) experience a path difference L, which yields

$$L = D \cdot \sin\theta \tag{3.5.2}$$

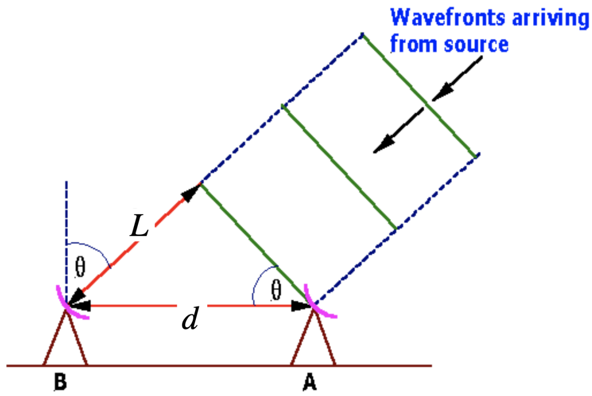


Figure 3.5.2: Inteferometry pattern for geometrical optics.

The two telescopes show maxima if

$$L = n \cdot \lambda \tag{3.5.3}$$

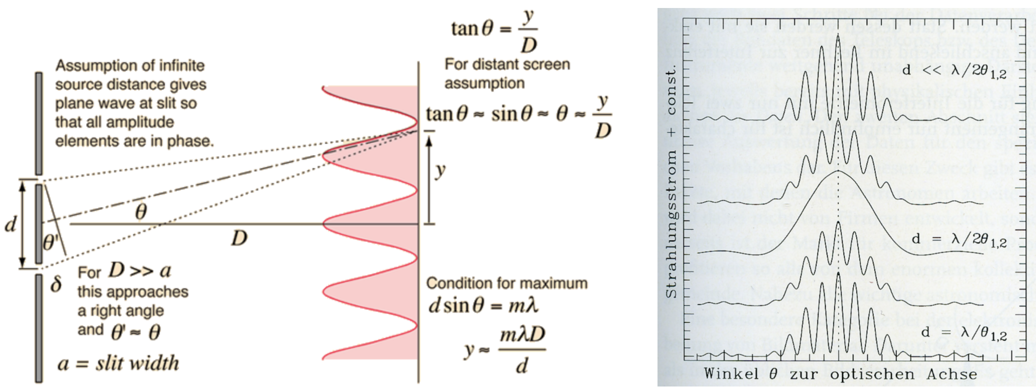
or minima for

$$L = \left(n - \frac{1}{2}\right) \cdot \lambda. \tag{3.5.4}$$

It is evident that the interferometry pattern follows the double slit principle.
Y is then the difference between 2 maxima:

$$Y = \frac{m\lambda r}{D} \tag{3.5.5}$$

The first 5 maxima all fit within the main maxima of a single aperture. When the two telescopes are fixed, so is D, and a maxima can be achieved if $\sin(\theta) = n\frac{\lambda}{D}$. It is then possible to determine the position of the source studying the interference pattern produced by the signals (θ), as an example using simply the earth's rotation which changes θ. However the interference pattern depends and changes with the baseline.



(a) Telescope interferometry.

(b) Interference pattern with changing baseline.

Figure 3.5.3: Basic principle of interferometry.

The improvement in angular resolution of a double aperture array with distance d with respect to a single aperture D is given by

$$\frac{\theta_{\mathrm{D}}}{\theta_{\mathrm{Deff}}} = \frac{\lambda}{d} \cdot \frac{2D_{\mathrm{eff}}}{\lambda} = \frac{2D_{\mathrm{eff}}}{D} \tag{3.5.6}$$

Only two telescopes are not enough as they are blind to angles $\theta_{12} >> \frac{\lambda}{2D}$, since the interference contrast disappears. This is why many telescopes at different base lines are necessary to offer a broad range.

Some known arrays:

- **Very Large Array (VLA)**: New Mexico, USA: 27 Antennas each 25m diameter ⇒ array baseline = 36 km ⇒ 0.1”
- **Very Long Baseline Interferometry (VLBI)**: different continents ⇒ 10^{-4}”
- **ALMA**:Atacama Large Millimetre Array,Chile , $D = 16$ km
- **Radioastron**: Russian commando but designed to be space based (future project).

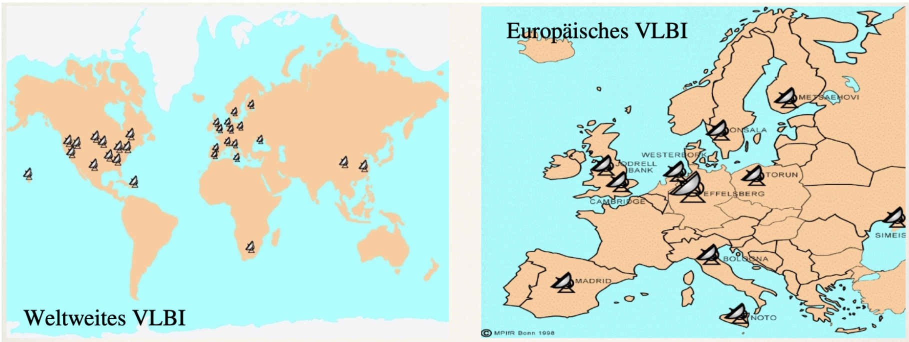


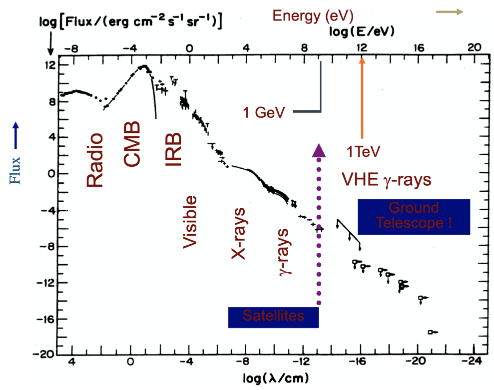
Figure 3.5.4: Global and European distribution of VLBI telescopes.

X-ray interferometry is not possible as of now, as it becomes increasingly difficult to hit a maxima with large base lines for shorter wavelengths. For optical, one needs high mechanical precision however there are also systematic problems or atmospheric fluctuations.

3.6 Infrared astronomy

Near Infrared (NIR) is, for the most part, still observable when ground based. However impinging effects such as by the atmosphere can still state a problem. This is why, besides ground based telescopes, there are also efforts to outsmart the issue by going beyond a substantial amount of the atmosphere, e.g. with the Stratosphären-Observatorium für Infrarot-Astronomie (SOFIA).

(a) SOFIA set-up.



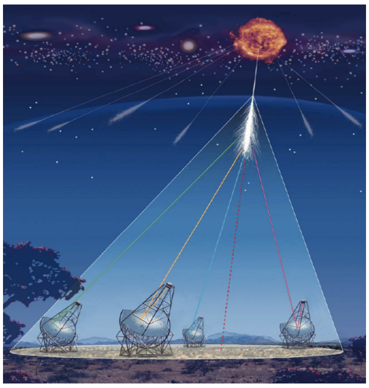
(b) Flux distribution for different wavelengths.

Figure 3.6.1: The SOFIA observatory (a) and the flux distribution for different wavelengths (b).

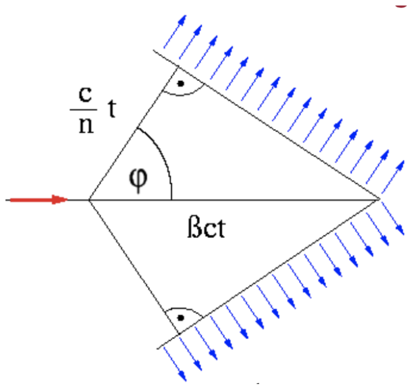
This plane based telescope is a DLR-NASA venture. Far infrared telescopes (FIR) are space based only. FIR telescopes such as IRAS, Spitzer, Hershel, ISO or JWST need to be cooled down to a few Kelvin to reduce the self inflicted noise by thermal radiation (liquid Helium). They are usually placed at the Lagrange point L2 in 1.5 million km from earth. JWST uses a proper multi-layer sun shield. IR is especially interesting as most objects of the re-ionized universe (early phase) appear strongly red shifted (to IR).

3.7 TeV astronomy

TeV describes energies above 10^{12} eV. Particles with these energies can not be measured directly in space as the incoming flux is too small. This would require a huge collective area. Again, it is technically not possible to place telescopes with the required collective area in space. However luckily, highly energetic particles penetrating the atmosphere create showers of secondary particles which cascade down to Earth's surface. These can be measured with so called *Cherenkov telescopes*.



(a) Cherenkov telescope principle.



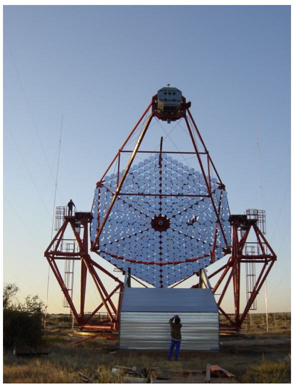
(b) Cherenkov radiation.

Figure 3.7.1: Cherenkov radiation detection.

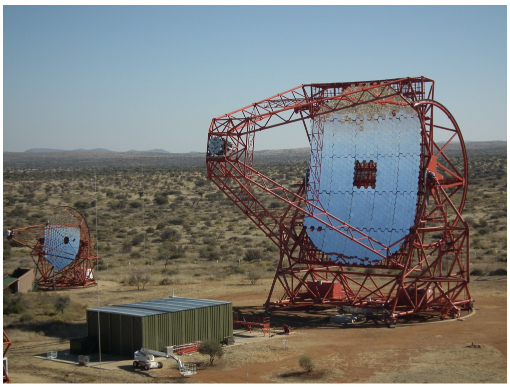
They can be either created by

- Pair production: electron/ positron
- Bremsstrahlung

These particles produced move at relativistic velocities which in turn allows them to radiate super luminally (Cherenkov UV light). These showers start at about 8 km altitude and last between 10 and 20 seconds. The dispersion angle is of the order of 1 to 2 degrees. This corresponds to a circle with radius of 120m. This means that an array of telescopes in necessary.



(a) H.E.S.S. front view.



(b) H.E.S.S. side view.

Figure 3.7.2: High Energy Stereoscopic System (H.E.S.S.) in Namibia.

The signals can be tracked and they are clearly distinguishable from other incident radiation. The only problem here is that the UV measurement requires the reduction and cancellation of all background. This is difficult as conditions basically need to be perfect, meaning: no weather infringements even if far away. They also require photo multiplier detectors (PMTs) rather than CCDs. Telescopes of this sort include the currently developed Cherenkov Telescope Array (CTA) and the existing High Energy Stereoscopic System (H.E.S.S.) in Namibia. Control segments, e.g. actuators for the HESS mirrors, were calibrated at the University of Tübingen.

Chapter 4

Solar system

Abstract. In this section we will look at our solar system, in particular at its composition and also at each planet individually. Although the other planets of our solar system are - on an astronomical scale - very close to us, there are still new stunning and exciting findings today. A recent example is the potential (!) habitability of Venus' atmosphere for microbes, which has been elaborated on due to anomalies in the atmosphere.

Keywords: Planets, greenhouse effect, tidal forces

Learning goals

- Name members of the Solar System.
- Name planets and their characteristics.
- What are differences between Solar System objects?
- What impacts on the energy system of a planet?
- What are tidal forces?

4.1 Definitions

The **solar system** is defined as a gravitationally bound system comprising the Sun and the objects that orbit it, either directly or indirectly. Our solar system currently consists of one star (the sun) with eight planets orbiting it, namely Venus, Mercury, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. Furthermore the solar system features 5 dwarf planets, namely Pluto, Ceres, Eris, Makemake and Haumea. However with decreasing size they are increasingly difficult to detect which hints at the assumption that there are many more potential candidates to be detected. Currently we have also listed 214 planetary satellites (another word for moons) in our solar system, most of them captured by Saturn (82) and Jupiter (79). Next on the list would be Uranus with 29 moons. It is not a coincidence that these planets have that many moons. Firstly, they have a very strong gravitational pull when compared to other planets like Earth. Secondly, they are thought to capture asteroids, for instance Phoebe, a moon of Saturn, is thought to be a former asteroid. In a sense, they protect earth from potentially harmful impacts. In addition, there are a ton of small solar system bodies: 829400 asteroids (millions to be detected larger than 1 km), 3592 Comets (billions to trillions to be detected), also there are many meteoroids and a lot of dust. Space, which may seem empty at first, actually is not that empty after all, if we consider all these bodies. On top of that, there is also interplanetary medium and solar wind, which includes radiation and magnetic fields.

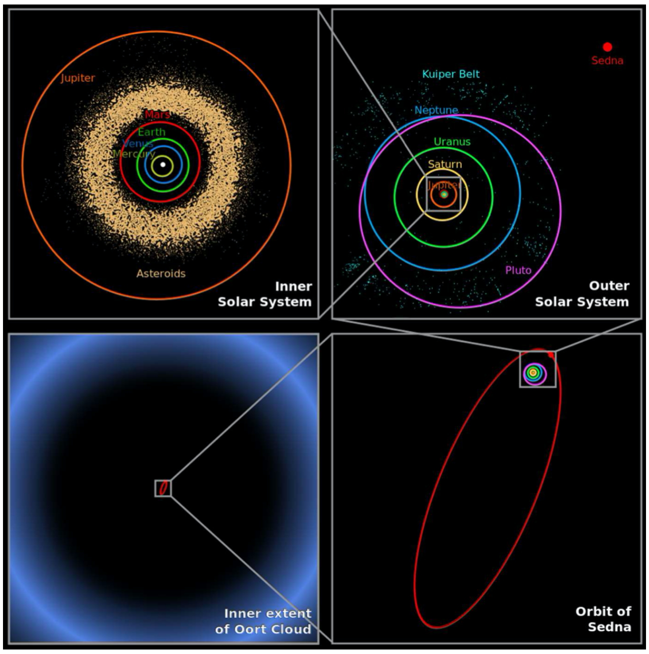


Figure 4.1.1: Solar system orbits.

Continuing our definitions, a **star** is a gaseous astrophysical object that, at least for some epoch of its evolution, undergoes nuclear burning in its core, so that its thermal heat production balances its own gravity (assuming hydrostatic equilibrium).

The definition of a planet actually involves some more conditions as specified by the*International Astronomical Union* (IAU) on August 26 in 2006. A **planet** is a celestial body that

- is in orbit around its local star
- has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape
- has cleared the neighborhood around its orbit

According to the IAU, a **dwarf planet** is a celestial body that fulfills the first two criteria of planets, but has not cleared the neighborhood around its orbit, while not being a satellite itself. Therefore the IAU downgraded the status of Pluto to that of a dwarf planet in 2006. Pluto shares its orbit with many objects of the Kuiper belt with Charon being the biggest neighbour with almost similar mass to Pluto itself. There are many more potential candidates to be accepted as dwarf planets by the IAU and maybe even more to be detected.
All other objects, except satellites, orbiting the sun (fulfilling the first criterion) shall be referred to collectively as **small solar system bodies**.

4.1.1 Spatial scales

In order to wrap one's head around the structure of the solar system, it is important to get a grasp of the vast scales that are involved. First, let us look at the following table that includes the distance from the planets to the Sun in AU (astronomical units, i.e. the distance from Earth to the Sun) and in meters with $1AU \approx 1.5 \cdot 10^{11} m$. All important conversion factors can be found in the appendix.

Planet	Semi-major axis [AU]	Semi-major axis [m]	Mean radius [10^3 km]	Mass [10^{27} g]
Mercury	0.39	$5.8343 \cdot 10^{10}$	2.4	0.33
Venus	0.72	$1.0771 \cdot 10^{11}$	6.05	4.9
Earth	1	$1.496 \cdot 10^{11}$	6.37	6.0
Mars	1.5	$2.244 \cdot 10^{11}$	3.4	0.64
Jupiter	5.2	$7.779 \cdot 10^{11}$	70	1900
Saturn	9.6	$1.436 \cdot 10^{12}$	58	570
Uranus	19	$2.842 \cdot 10^{12}$	25.4	87
Neptune	30	$4.488 \cdot 10^{12}$	24.6	100

Table 4.1: Solar system: distances to the Sun.

We can see in table 4.1 that the distances between the planets increase strongly for the outer planets. For a nice visualization of the spatial scales of the solar system, consider to download the „Exoplanets“ App developed by Hanno Rein or use one of the many available online tools. In addition it is worth mentioning that the combined mass of all celestial bodies in the solar system excluding the sun, amounts to approximately 1%.

Looking at the bigger picture, we must also mention belts in the solar system. In fact, there are two of them. One is the asteroid belt in between Mars (~ 1.5AU) and Jupiter (~ 5.2AU) which includes the dwarf planet Ceres. On the other hand, there is the famous Kuiper belt behind Neptune ($30-50$AU) which hosts the dwarf planets Pluto, Eris, Makemake and Haumea. Zooming out even further reveals two more interesting objects. One is Sedna: a body with an eccentric orbit ($e \approx 0.85$, perihelion: 80AU, aphelion 900AU and semi-major axis 500AU). Even further out, there is the Oort cloud with comets beyond $50,000$AUwith a combined total mass of 5 Earth masses. All of which can be seen in 4.1.1.

Finally, let us look at the extension of the bodies in the solar system. For that, we look at the following table, presenting the approximate radii of many celestial bodies.

Object	Radius [km]
R_{sun}	$6.957 \cdot 10^5$
R_{Jupiter}	$7.1492 \cdot 10^4$
R_{Earth}	$6.3781 \cdot 10^3$
R_{Planets}	$[2.4, 70] \cdot 10^3$
$R_{\text{DwarfPlanets}}$	[500, 1200]
R_{Moons}	<2700
$R_{\text{Asteroids}}$	[0.001, 300]
R_{Comets}	< 200
$R_{\text{Meteroids}}$	< 0.001

Table 4.2: Solar system: distances to the sun

Discovery and exploration

A quote of NASA's outreach sums up why we should discover and explore our solar system:

> "The Solar System — our Sun's system of planets, moons, and smaller debris — is humankind's cosmic backyard. Small by factors of millions compared to interstellar distances, the spaces between the planets are daunting, but technologically surmountable." (NASA, Solar System Exploration Roadmap 2006)

There are several ways to collect information about the Solar System: Telescopes (Earth-bound, space telescopes), space missions, laboratory work / material science experiments and not to forget, the development of theories and the application of numerical simulations.

4.1.2 Energy balance of planets

Almost no other variable in space is as important to the outcome of celestial bodies as their ambient temperature. It determines fusion in stars and habitability on planets. Naturally a multitude of factors contribute to the local temperature system. The biggest driver, when it comes to temperature input in the Solar System, is the Sun.
The solar constant determines it's energy flux and is given as S = 1370 $\frac{\mathrm{W}}{\mathrm{m}^2}$ or more generally

$$S(r) = S \cdot \left(\frac{r}{1\,\mathrm{AU}}\right)^{-2} \tag{4.1.1}$$

It strongly depends on r, the distance from any object to the Sun and is normalized to 1AU. Further of importance is the *Albedo value* which denotes the reflectivity or ability to absorb thermal energy from radiation. A=1 for white surfaces with perfect reflection (no energy intake), A=0 for black surfaces (complete absorption). Earth has an Albedo of 0.31 which among other things depends on the amount of ice and clouds. When in equilibrium, the amount of energy absorbed by a planet at distance r is given by

$$\pi R^2 \cdot (1-A) \cdot S(r) \tag{4.1.2}$$

equals the (Black body) emission declared by Stefan Boltzmann. In this case

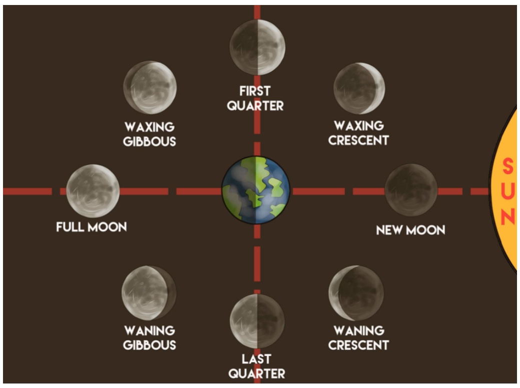
$$4\pi R^2 \cdot \sigma_{\mathrm{sb}} \cdot T_{\mathrm{eff}}{}^4 = 4\pi R^2 \cdot F = \pi R^2 \cdot (1-A) \cdot S(r) \tag{4.1.3}$$

Other effects include inner heat (gravitational quenching inside Jupiter and Saturn) or the greenhouse effect which denotes the ability of the atmosphere to stop planetary *re-radiation* (in IR). The "goodness" of which obviously depends on the constituents (CO_2) and thickness of the atmosphere. A planets temperature also depends on night and day, so rotational speed is also to be considered.

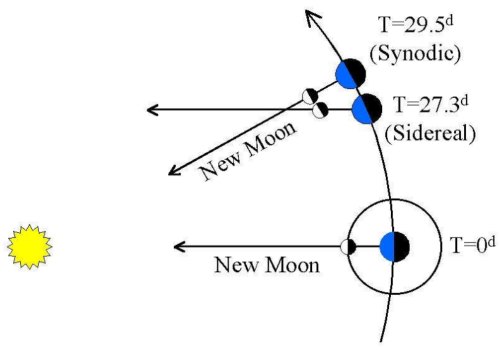
4.1.3 Lunar specs

The Moon is out closest compatriot and a distance of 384 000km which is close to 60 Earth radii. The Moons periapsis is 362 000 km and its apoapsis 405 000 km with almost no eccentricity making the moon appear 3' larger when it is at its periapsis. Its mass is only 1/81 of Earth while its radius is 27% that of Earth. This means that the joint center of gravity, around both objects orbit is still within Earth. In terms of timing we have:

- siderial month: time that it takes for the moon to orbit the earth 27,3 d
- synodic month: 27,3 d + earth moving around sun. So new moon to new moon is 29.5 days.



(a) Lunar phases.



(b) Synodic and Siderial timing.

Figure 4.1.2: Moon characteristics.

A solar eclipse is achieved, when the Moon is located between Sun and Earth. A lunar eclipse means the Moon is in Earth's shadow.

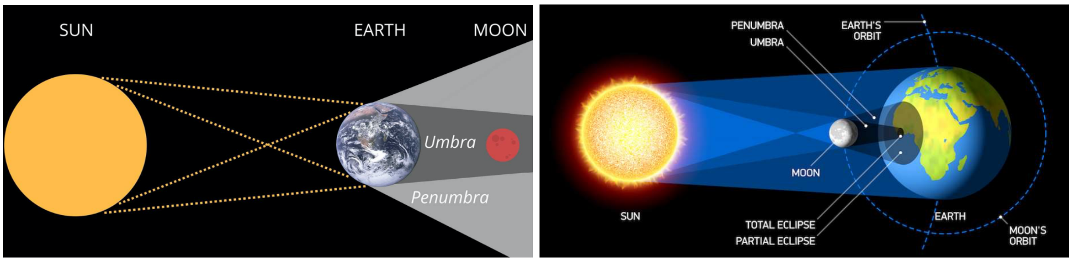


Figure 4.1.3: Lunar and Solar Eclipses.

The Moon features approximately 80% Terrea, meaning bright, high-lying areas, scattered with craters, which are linked to meteorite showers. 20% are Maria, meaning dark and low-lying areas. It is thought that the Moon came into existance around the same time Earth did which was $4.5 \cdot 10^9$ years ago.

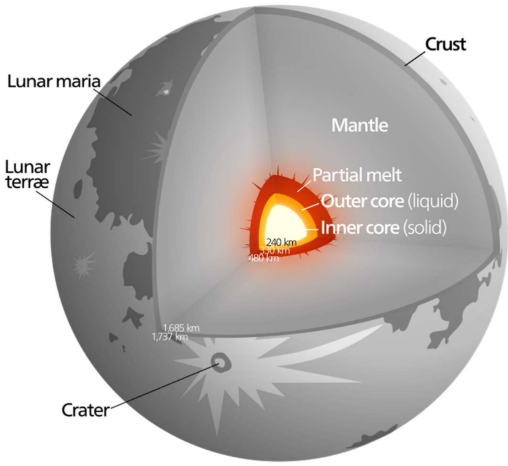


Figure 4.1.4: Moon interior.

Formation theory suggests that the Moon was created by Earth crashing into a smaller (Mars size) planet, as the Moon consists in parts of the same materials found in Earth's mantel. Today the Moon has a surface gravity of 1/6 from what we have on Earth meaning it can not hold on to an atmosphere. Also it indulges extreme weather with temperatures ranging from -130°C to + 130°C.

4.1.4 Gravitational interaction

The side of an object closer to moon, such as like on Earth, will be attracted stronger via gravity in comparison to the part further away due to non zero size and mass of the object. This means the object can be deformed or even brake. This is called **tidal disruption effects**. On Earth this means that the moon inflicts 2 high tides a day. The side facing the moon witnesses the water being pulled towards the moon more strongly, creating a bulge every 12h 25min. Simultaneously the side facing away from the moon allows the less strongly attracted water to escape slightly, resulting in the second bulge. You could say, Earth rotates beneath the tides which are approximately 2.5h advanced (bulge not pointing directly towards moon). The bulges therefore inflict a torque (tidal friction), which means that the moon is slowing down Earth's rotation. As the angular momentum of the system must be conserved, the distance between moon and earth increases by approximately 4cm per year.

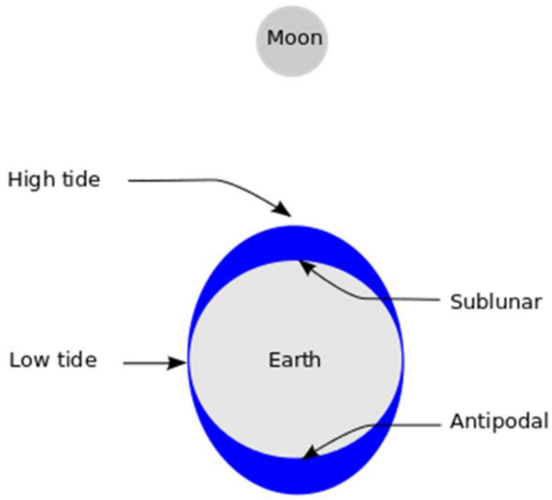


Figure 4.1.5: Tidal forces.

Therefore the length of the months increases. Equilibrium will be reached when Moon and Earth face the same side of each other, or 1 day equals 1 synodic month. This will happen in roughly 10^9 yr. For a more extensive explanation, see chapter 4.3.

4.2 Planets

We will now look at each planet individually and also at the different categories for classifying planets.

Rocky planets

The characteristics of rocky planets are that they have a *solid surface* and gaseous atmosphere of negligible mass. They usually feature *craters* which they most likely collected during the *late heavy bombardment* phase (such as the moon). Naturally, planets with active and dynamical surfaces, i.e. via volcanic activity allow for craters to disappear over time (like Earth). They can feature *planetary satellites* like Earth's moon and the two moons of Mars.
Finally they can feature *potential magnetospheres*. The origin of magnetospheres is linked to the liquid iron core of a planet which functions as a huge dynamo which creates the magnetic field, due to rotation with respect to the upper layers of the planet.

It is worth mentioning that the magnetosphere is the lone protector of a planet in regard to the Sun's solar wind. Since the surface area to volume ratio increases with decreasing size for spherical objects, smaller planets are likely to cool quicker. This in turn has solidified Mars' once liquid core over time, resulting in a reduction of the magnetosphere and ultimately having its atmosphere stripped due to less protection via the magnetosphere. Without an atmosphere, maintaining a balanced temperature household is close to impossible in the hostile environment which is space. Consequently Mars is the deserted world we see today. Regarding the internal structure of rocky planets, we can say that in general they have a radial extent silicate mantle which is equal to or larger than the radial extent of the iron core, which has sizes of around 1800 to 3500km. On top, there is a crust layer which is between 5 and 125km thick.

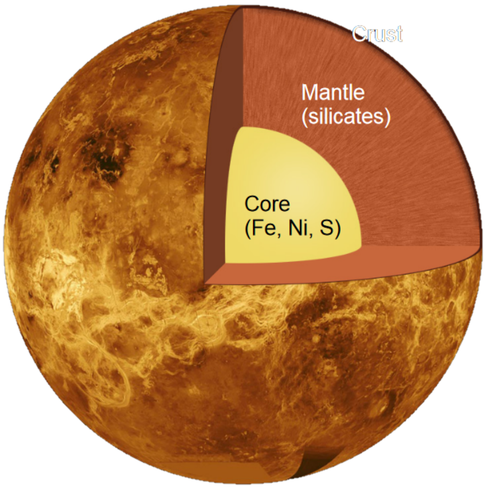


Figure 4.2.1: Internal structure of rocky planets.

- core consists of solid or fluid Fe, S, Ni
- mantle of the core is made of viscous, molten SiO_4 rich rock. Rising material in the mantle causes volcanoes. Mantle dynamics can drive plate tectonics and thus also lead eventually to volcanoes and earthquakes.
- Crust is thin ≈ 100km silicate rock

Rocky planets: Mercury

- highest ellipticity e=0.2
- 3:2 spin orbit resonance meaning 3 rotations per 2 revolutions. In this constellation the time between two sun rises is 2 years.
- almost no atmosphere
- geologically active for billions of years
- partially liquid core, hints of weak magnetic field
- surface similar to Moon highlands with craters, but no Maria
- no moons
- small and is scorched by the sun
- temperatures range from 330°C to 430°C during day time to -180°C during night time
- Space Missions (selected)
  - Messenger (NASA orbiter, 2011-2015)
  - Bepi Colombo (ESA & JAXA with 2 orbiters, 2018 lift off and arrival 2025)

Rocky planets: Venus

- Counter-rotating (slowly: 243 days for a year) with respect to Earth's spin
- Extremely dense atmosphere at 96 bar (96.5% CO_2, 3.5% N, ...)
- Strong greenhouse effect with opaque clouds in 40-70km altitude raining sulfuric acid; hottest surface planet due to strong greenhouse effect T > 460°C
- wind speed of clouds at roughly 300 km/h. Probably linked to the thermal activity
- Active volcanism, no plate tectonics, no internal magnetic field
- No moons
- Second brightest object in the night sky (after the Moon)
- 2 continents as inferred from surface structure analysis
- Space Missions (selected)
  - Mariner 2 (1962)
  - Venera 1-16 (Lavochkin Soviet Union)
  - Venera 7: first landing on another planet in 1970
  - Mariner 10 (NASA, 1973)
  - Vega 1+2 (Lavochkin, Soviet Union, 1984)
  - Magellan (NASA, 1989)
  - Venus Express (ESA, 2005)
  - Akatsuki, IKAROS, and Shin'en (JAXA, 2010)

Figure 4.2.2: Venus landscape.

Rocky planets: Earth

- Largest terrestrial planet, densest planet
- Geology: volcanism, plate tectonics, magnetic dynamo, liquid water
- Axial tilt of 23° → Seasons
- Second highest moon:planet mass ratio (after Pluto and Charon)
- (Intelligent) life

Rocky planets: Mars

- Outermost terrestrial planet
- Two moons: Phobos and Deimos
- Axial tilt of $25° \rightarrow$ Seasons
- Geology: volcanism, inactive plate tectonics, no magnetic field
- Thin atmosphere $\rightarrow$ low pressure $\rightarrow$ nearly no liquid water
- red colour is due to iron oxide with white poles covered in ice
- it features the biggest ridge in the Solar system **Vallis Marineris**: 4000km long, 200km wide and 7 km deep. For comparison, the Grand Canyon in Arizona is about 800 km long and 1.6 km deep. Simultaneously it has the highest mountain in the Solar system **Olympos mons** at 24km height and 600 km width. Again, for comparison Mount Everest is 8864 m high.
- Space Missions: ... many

Dark, narrow streaks on Martian slopes such as those seen at Hale Crater are inferred to be formed by seasonal flow of water on contemporary Mars. There have been many rovers on Mars, such as in the Mars Pathfinder mission (1997), in the Mars Exploration Rover (MER) mission sending the rover called "Spirit" (2004 - 2010) as well as Opportunity (2004 - June 2018). In the Mars Science Laboratory (MSL) mission the rover Curiosity (2012 - present) has been sent to Mars and it is still operating there today.

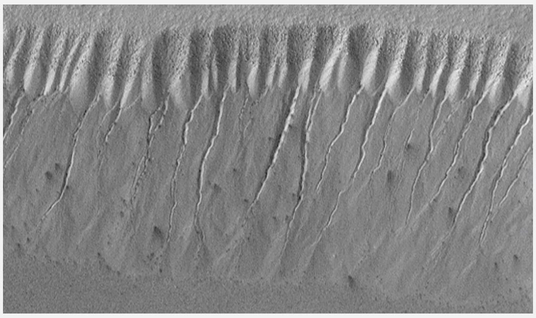


Figure 4.2.3: Water on Mars ?

Giant planets

Besides the rocky planets, there are also the giant planets. Unlike rocky planets, they have *no solid surface*. The giant planets can be divided into two subcategories. First there are the *gas* giants which include Jupiter and Saturn, secondly there are the *ice* giants like Uranus and Neptune. Most of them have *ring systems*. The rotation period is high and lies between 10 an 18 hours. Since they have no solid surface, they become oblate due to centrifugal forces. Also, the *atmospheres* are rather interesting: they do not rotate uniformly, but they rotate faster at the poles than on the equatorial height. This is why non uniform wave fronts show frictions patterns (such as on Jupiter). The compositions are close to pre-solar nebulae, i.e. 75% hydrogen, 25% helium by mass. In the case of gas giants the atmosphere predominantly features hydrogen, helium and traces of metals in it. Ice giants on the other hand also harbour, helium, and traces of metals, but also a lot of ice. Both categories also have a *magnetosphere*. For gas giants, there is a dynamo effect due to currents in the interior metallic hydrogen layer. The origin of the fields on Ice giants is not yet fully understood.

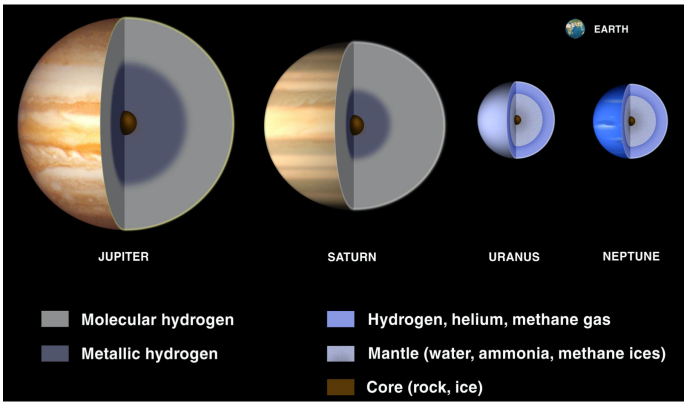


Figure 4.2.4: Gas and Ice giant interior.

- Core: silicate and ice further out
- Mantle: liquid metallic hydrogen for Saturn and Jupiter. This inflicts a magnetic field. Hence is why Uranus and Neptune are called Ice giants, because their mantle consists predominantly of water ice.
- Crust: no crust -> gaseous hydrogen

Giant planets: Jupiter

- Largest and most massive planet
- Jupiter's mass is 2 times the mass of all the other planets combined. It influences the asteroid belt as well as the orbit of the Greeks, Hildas and Trojans.
- 53 + 26 = 79 moons
- Galilean moons: Io (yellow due to high amounts of sulphur & SO_2. It has the most active vulcanism due to the gravitational heating of Jupiter), Europa (like our Moon and expected to have an ocean under a thick crust of ice.), Ganymede (size of mercury and half its mass, thus only moon with magnetic field), Callisto (more like snowball) (Galileo Galilei 1610)
- Fast rotator (rotation 10h) $\rightarrow$ oblate
- "Great Red Spot" (cyclone) wandering over Jupiter's visible surface
- Strongest magnetosphere of all planets
- Kelvin-Helmholtz-Contraction (2 cm/yr) $\rightarrow$ release of gravitational energy

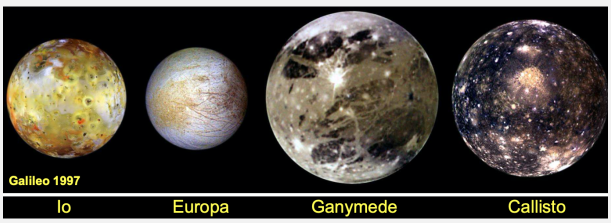


Figure 4.2.5: Galilean Moons.

Giant planets: Saturn

- Ring system (remark: all giant planets have one, however the density is usually much smaller): ice, rocky debris, dust
- Fast rotator (rotation 10h) → oblate
- 53+29 = 82 moons
  - Famous moon Titan (maintains an atmosphere)
  - Famous moon Enceladus (harbors icy geysers from liquid salt water)
  - Only planet with two moons in co-orbital motion
- No Trojan asteroids (in contrast to all other planets except Mercury)
- Magnetosphere (5% of Jupiter's magnetic field)
- Features lakes of fluid methane
- Most famous / recent mission: Cassini-Huygens (NASA / ESA / ASI)
  - Launched on October 15, 1997
  - In orbit around Saturn on July 1, 2004
  - Huygens (ESA) landed on Titan on January 14, 2005
  - Controlled crash on September 15, 2017

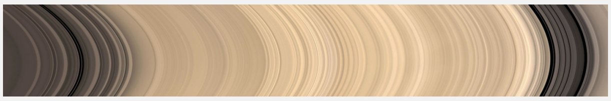


Figure 4.2.6: Galilean Moons.

Giant planets: Uranus

- Axial tilt of rotation very close to the ecliptic at 90° → by far the most unusual orientation (previous collision)?
- Just visible with the naked eye at mag = 5.6 (human eye mag = 6)
- 27 moons
- Atmosphere
  - Featureless but coldest atmosphere of all planets 50 K
  - Cyan color (in the visible) due to 2.3% methane, CH4 (by volume)
- Even lower heat flux than Earth → collision or internal heat flux barrier
- Magnetic field tilted by 60° from rotation axis; center shifted by 1/3 of the planetary radius towards southern rotational pole
- Mission: only a single fly-by of Voyager 2 (NASA) in 1986

Giant planets: Neptune

- Although is not visible with the naked eye it can be predicted due to multi body system pertubations. It interacts with the Kuiper belt as well as with Pluto who is crosses orbits with.
- Proposed by Alexis Bouvard (due to irregular orbit of Uranus) in 1821; Predicted by John Couch Adams and Urbain Le Verrier in 1845; Observed by Johann Gottfried Galle in 1846
- 14 moons
- Axial tilt of $28°$ $\rightarrow$ Seasons
- Outer atmosphere temperature 55 K > Uranus' temperature
- Magnetic field tilted from rotation axis and offset from center
- border of planetary system reached. Further out: Pluto and other Trans-Neptune Objects (TNOs) compose the Kuiper belt
- Gravitationally interacting with the Kuiper belt
- Mission: only a single fly-by of Voyager 2 (NASA) in 1989
- "Great Dark Spot" (anticyclone detected in 1989), "bright smudge", and "Small Dark Spot" (southern cyclonic storm in 1989); lifetime of months

Now that we have gone into detail about every single planet, we turn towards out next objects of interest which are the dwarf planets.

Dwarf planets

The dwarf planets in our solar system are Pluto, which has 5 moons, Ceres with no moons, Eris, Makemake and Haumea. Ceres is located in the asteroid belt, the others in Kuiper belt. Up to date (2020), only missions to Ceres and Pluto have been completed. Dwarf planets have a *highly eccentric* orbit in general with *high inclination*.

Let us end the list previously started by listing remarkable facts about Pluto and Ceres only.

Dwarf planets: Ceres

- Only dwarf planet outside the Kuiper belt
- Classified as an asteroid until 2006
- Smallest object confirmed to be in hydrostatic equilibrium
- "Failed Planet" (due to vicinity of Jupiter?)
- No moon •Surprising: No large craters $\rightarrow$ icy layer below the surface
- Contains liquid water $\rightarrow$ Potential for life?
- Mission: Dawn (NASA) in 2015

Dwarf planets: Pluto

- Classified as a planet until 2006
- From 1979 to 1999, Pluto was actually closer to the sun than Neptune
- 5 moons: Charon, Nix, Hydra, Kerberos, and Styx
- Highest M_{Moon} / M_{Planet} mass ratio (Pluto - Charon) at almost 1:1 which means that the center of mass of the system lies outside Pluto.
- Mission: New Horizons (NASA) in 2015

4.3 Tidal forces

We will now introduce gravity, centrifugal forces and the rotation movement step by step in order to illustrate how forces on a single body (e.g. on a planet) are not uniformly distributed, but rather have different strengths on different areas.

Starting with gravity, we get the following sketch.

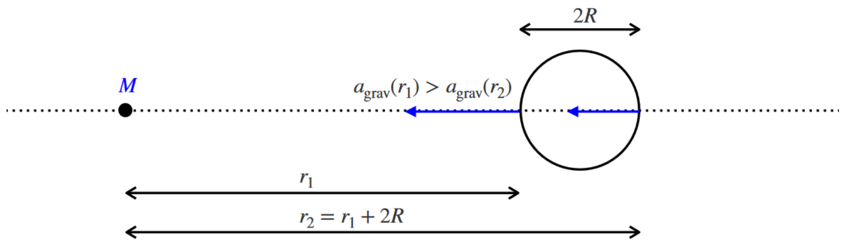


Figure 4.3.1: Tidal forces: Gravity

From this illustration and from the fact that gravity depends on the distance r, it becomes clear that there must be a different gravitational force pulling on the front of the planet and on the back of it. Depending on the distance r_1 or r_2 to the central mass M, the size R of the affected object, and its material properties change. The object can be distorted or even break apart, which is called "tidal disruption". Considering the centrifugal force, we obtain the following, modified situation:

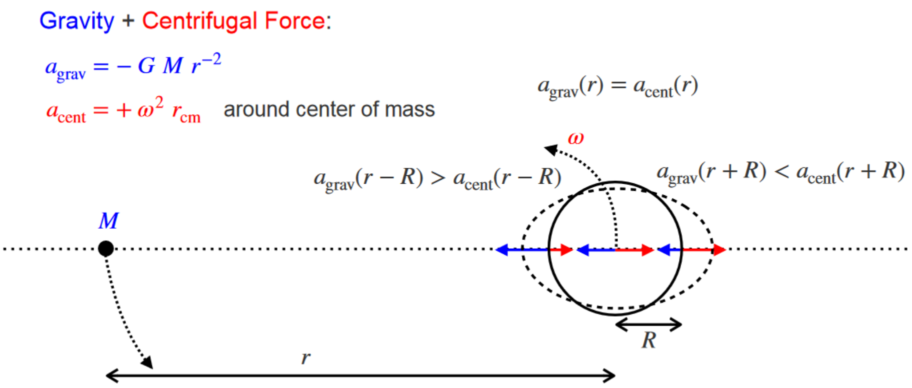


Figure 4.3.2: Tidal forces: Gravity + centrifugal force

If we assume a homogeneous mass density of the undisturbed planet, then

$$a_{\text{grav}}(r) = a_{\text{cent}}(r) \tag{4.3.1}$$

holds at the center of mass of the planet at any time, even with distortion. Also, the centrifugal acceleration due to the centrifugal force is given by

$$a_{\text{cent}} = \omega^2 r^2. \tag{4.3.2}$$

Thus, the inside facing part of the planet (smaller r) experiences a weaker centrifugal force than the outside facing side (larger r). Taking the gravitational force into consideration, this leads to further distortion and the object becomes elliptical with two bulges. In figure 4.3.2 the left-hand side of the planet is dominated by the gravitational force (blue) and the right-hand side by the centrifugal force which points in the opposite direction. In the center, the forces are in balance. In this final step, we want to introduce the rotation of the celestial body itself.

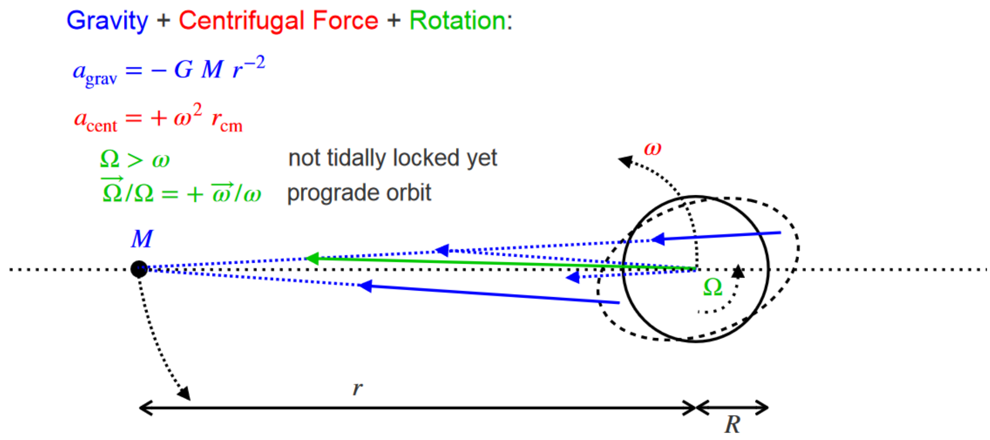


Figure 4.3.3: Tidal forces: Gravity + centrifugal force + rotation

This effect leads to shifted bulges due to drag forces. This in turn may result in tidal heating of layers of the planet. Also the gravitational net torque changes due to offset of the massive bulges. Furthermore, the spin of the object slows down over time. If the object is a fast rotator, a slow satellite rotating in propagate direction (in the same direction as the rotation around the central mass), this results in tidal locking, i.e. there is only one side of the body facing the central mass for all times and one facing away from it. If, on the other hand, the object is a fast satellite, a slow rotator and has a propagate orbit, it also can be tidally locked, but also disrupted.

The following table sums up all the effects in every case discussed.

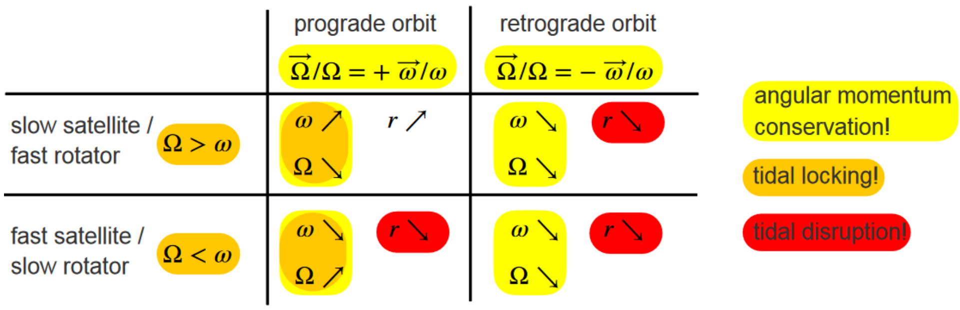


Figure 4.3.4: Tidal forces: Summary

4.3.1 Roche limit

In this context it is also clear to see how rings form around very massive planets or stars. For that we define the Roche limit as the the smallest distance of a celestial body below which objects around it are gravitationally disrupted by its tidal forces. These were discussed in the context of Roche lobes of stars in section 2.3. At the Roche limit

$$d_{\text{Roche}} \propto \left(\frac{M_{\text{primary}}}{\rho_{\text{secondary}}}\right), \tag{4.3.3}$$

the forces that hold the secondary mass together are overwhelmed by the tidal forces of the primary mass. Its "pulverized" leftovers can then orbit the star at the Roche limit. One may ask why there is a Roche limit at all, i.e. why the difference in net force ΔF becomes larger at all. The reason for that is quite simple. We can just look at the gravitational force, which is $F_{grav} \propto \frac{1}{r^2}$. Thus, for smaller distances r, the difference in gravitational force at small distances (red points in figure 4.3.5) becomes much more significant than for larger distances (blue points in figure 4.3.5).

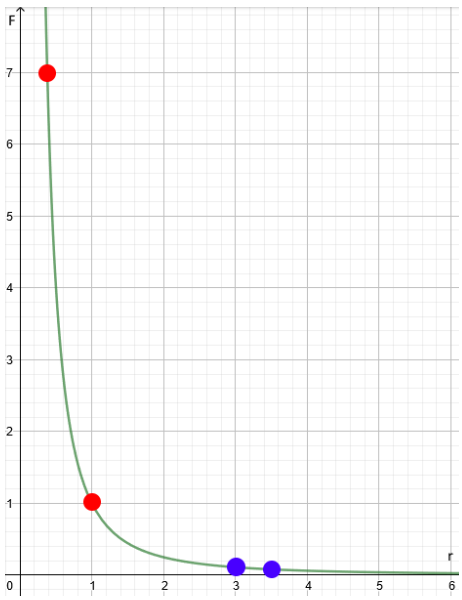


Figure 4.3.5: Tidal forces: Roche limit. The plot shows $F(r) = \frac{1}{r^2}$. Red points and blue points are both $r = 0.5\,\text{a.u.}$ apart.[1]

4.4 Planetary Satellites

> "Planetary satellites (often called "moons") are not to be confused with man-made satellites (such as weather and communications satellites) in orbit around the Earth. Planetary satellites are small bodies in orbit about a planet (actually the planet's system barycenter). Probably the best known planetary satellite is Earth's moon. Currently, we know of no satellites in orbit around Mercury or Venus. All the other planets have at least one known planetary satellite."

- Ryan S. Park, NASA.

[1]The term “a.u." here should not be confused with “A.U.". Latter stands for the astronomical unit, which is a well defined length. “a.u." on the other hand stands for “arbitrary units" and indicates that the exact order of magnitude is not relevant for the discussion.

Speaking of planetary satellites, it is worth to consider the "dancing moons of Saturn". One moon is Epimetheus with an orbit at approximately $152,010/90 \pm 10$km, which is about 0.3AU. It has a radius of about 60km and a mass of $5.3 \cdot 10^{20}$g.
Janus, the second moon of interest has a orbital distance of approximately $152,060/40 \pm 10$km with a radius of around 90km and a mass of around $19 \cdot 10^{20}$g, which is 3.6 times as much as his fellow moon Epimetheus. Epimetheus and Janus are on a co-orbit (orbital difference ≤ 50km). Due to their different orbital speed, they 'meet' roughly every 4 years.
The gravitational attraction of the two moons leads to a (de-)acceleration of their orbital velocity, accompanied by a shift in their orbital radius according to Kepler's third law of planetary motion. That is to say, that the moons do not only feel the gravitational pull of Saturn itself, but also the one of the other moon. The following illustration shows these "dancing moons", namely Epimetheus and Janus.

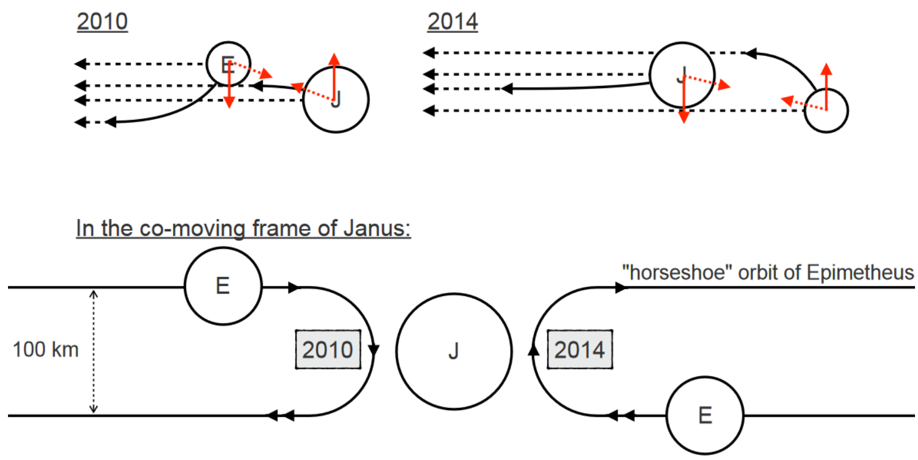


Figure 4.4.1: Dancing moons of Saturn

After they meet, they swap orbits in a 100 days lasting maneuver. In the frame co-moving with Janus (figure 4.4.1), Epimetheus is on a so-called "horseshoe orbit" (related to the circular restricted three-body problem). The closest distance of the two moons is $15,000\text{km} > 100R_{Epimetheus/Janus}$, but they never pass each other.

Space missions to analyse planetary systems include

- Pioneer 10 in 1973 and Pioneer 11 in 1974
- Voyager 1 and 2 in 1979
- Galileo (NASA) from 1995 to 2000 •New Horizons in 2007

4.4.1 Asteroids

Asteroids are small but usually bigger than 1km across (compared to all previously discussed objects), rocky bodies that orbit the Sun. There are approximately 500 000 objects mainly located within the asteroid belt between Mars and Jupiter and as Trojans in co-orbital motion with planets (except for Mercury and Saturn). Sometimes one includes trans-Neptunian objects (TNO) (aka minor planets). Many other TNO's are probably dwarf planets by definition (remember: "spherical"). Else, they belong to the class of asteroids. It becomes exceedingly difficult to decide. An example of this sort is the asteroid Arrokoth. The largest asteroids are "(4) Vesta" and "(2) Pallas" with radii up to 260km, where the numbers in brackets are the so called "asteroid discovery number". The smallest asteroids are only about a few meters in diameter. The ration of number density to size follows a power law. Even asteroids can have moons or satellites too. The composition of an asteroid and its interior structure can vary from that of a (dwarf) planet to uniform / undifferentiated structures. Many missions have discovered asteroids and investigated their properties, such as: Galileo (NASA) imaged (951) Gaspra and (243) Ida in 1991 and 1993, NEAR Shoemaker (NASA) in orbit around (433) Eros, landed in 2001, and many more.

4.4.2 Comets

Comets are small, rocky-icy bodies orbiting the Sun (basically dirty snowballs with silicates). They are mainly located within the Kuiper belt behind Neptune with a rather short period of $P < 200d$ and within the Oort' cloud where long-period comets orbit with $P > 200d$ in isotropic orbits.
The Structure of comets can be simplified as follows: They have a nucleus, which is solid with a rocky-icy core of $R \in [1, 50]$km. The coma is a dusty, gaseous atmosphere with an extension of around $R \approx 10^5$km. Last but not least, comets have tails of a length up to 10^6km and consist of ionized gas (pointing away from the Sun) and dust (dragged along).
Whenever they approach the Sun on its highly elliptical orbit, the water ice evaporates and creates a gaseous envelope/halo. Regarding the missions, Giotto was one of the first which was a fly-by by ESA of 1P/Halley in 1986 made the first imaging of a nucleus. Many more followed by the NASA and ESA, even with sample returns. One of the latest achievements in this area was the exploration of the comet 67P/Churyumov-Gerasimenko with the ESA mission ROSETTA and its lander PHILAE.

4.5 Solar wind

The sun frequently ejects energy in form of particle showers which can be linked to flairs or mass ejections. These are distributed via charged particles such as electrons, protons or He-nuclei which travel at approximately 800 $\frac{km}{s}$. They impinge on earth in numbers of 5-10 per cubic cm. They represent the suns mass loss of roughly 2-3 10^{-14} solar mass per year. This is comparatively low. Earths magnetic field, which shields us by rerouting the particles (Aurora), is continuously deformed by the solar winds pressure.

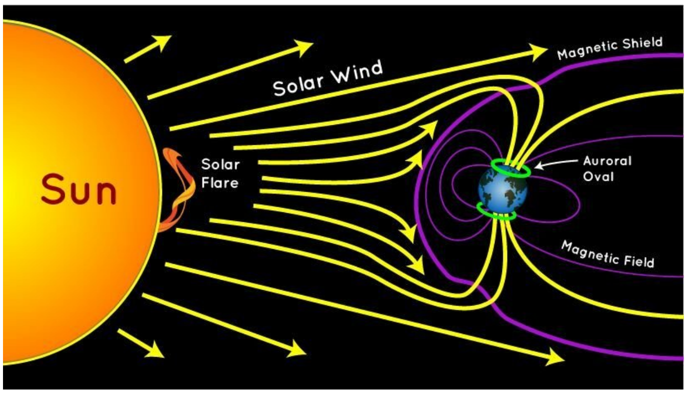


Figure 4.5.1: Deformation of Earth's magnetic field by solar wind.

4.5.1 Others

Besides the objects discussed above, there are many more objects present in space. For instance in our solar system there are interplanetary media. The composition of the interplanetary medium is dust, cosmic ray and solar wind. The density follows the distribution

$$\rho_{IPM} \propto (\vec{r} - \vec{r_*})^{-2}, \tag{4.5.1}$$

where $\vec{r_*}$ denotes the position vector of the Sun. We can see, that the density decreases in a quadratic relation. The solar wind can be neglected at between 110 and 160AU. Beyond this point the interstellar medium (ISM) dominates. But that is not all, magnetic fields and radiation also fill interstellar space. We can observe the solar wind when it interacts with comets and the aurora of the earth.
Even at the outer edge of the solar system, there is still something happening: The heliosphere is the border where the solar wind impact becomes stronger than the pressure of the interstellar medium. The termination shock describes, as the term suggests, the location, where the solar wind becomes subsonic. The heliosheath is the turbulent outer edge of the heliosphere and the heliopause the equilibrium region between the heliosphere and the interstellar matter. From here on out there are no further changes - the "no mans land" of our solar system if you want to call it that.

Chapter 5

Fundamental properties of Stars

Abstract. Introduction of the general properties used to describe radiation from stars: luminosity, flux, intensities, emissivity and absorption. We discuss the radiation transport equation, and the black body spectrum. In addition we also introduce the Stefan-Boltzmann law. Finally we touch up on magnitudes and colours.

Keywords: Flux, radiation transport, black body emission, magnitudes

Learning goals

- How are the Flux, specific intensity and luminosity defined? What is the physical meaning of these quantities?
- What does radiation transport mean? How is the intensity modified by emission and absorption?
- What are the main features of the black body spectrum?
- What are apparent and absolute magnitude? How are these quantities changed by extinction?
- What do colors denote in astronomy?

As you have learned in the introductory courses on quantum mechanics, photons have a dual nature, one of the particle and the other as a wave.[1]
Wavelength and energy of a photon are given as:

$$\lambda = \frac{c}{\nu} \quad , \quad E = h\nu. \tag{5.0.1}$$

From the above equations, momentum can be written as:

$$p = \frac{h\nu}{c} \tag{5.0.2}$$

Single photon counts are important in astrophysics, but ultimately we are interested in macroscopic quantities concerning radiation.

Region	λ(Å)	λ (cm)	Frequency (Hz)	E (eV)
Radio	$> 10^9$	> 10	$< 3 \times 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \times 10^9 - 3 \times 10^{12}$	$10^{-5} - 0.01$
IR	$10^6 - 7000$	0.01-7 $\times 10^{-5}$	$3 \times 10^{12} - 4.3 \times 10^{14}$	0.01-1.5
visible	7000-4000	$7 \times 10^{-5} - 4 \times 10^{-5}$	$4.3 \times 10^{14} - 7.5 \times 10^{14}$	1.5-3
UV	4000-10	$4 \times 10^{-5} - 10^{-7}$	$7.5 \times 10^{14} - 3 \times 10^{17}$	$3 - 10^3$
X-Rays	10-0.1	$10^{-7} - 10^{-9}$	$3 \times 10^{17} - 3 \times 10^{19}$	$10^3 - 10^5$
Gamma Rays	<0.1	$< 10-9$	>3×10^{19}	$> 10^5$

Table 5.1: Ranges and characteristics of electromagnetic spectrum.

5.1 Radiation

5.1.1 Basic Definitions

In this chapter, we will review the most fundamental notions of radiation, namely the Flux, total flux, the intensity and their connection, the energy density, momentum flux density and finally the luminosity.

The Flux

Let's assume that in a time interval dt a total energy dE is received by an element of area dA (it could be a detector or an element on the surface of the star!) perpendicular to the direction from which the radiation arrives. **F can change depending on the orientation of the element.**

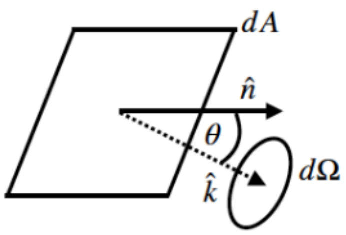


Figure 5.1.1: Flux through a surface element

The flux F and its units are defined as:

$$F = \frac{dE}{dAdt} \quad , \quad [F] = \frac{\text{erg}}{\text{cm}^2\,\text{s}} \tag{5.1.1}$$

With *ergs* being an alternative unit for Energy (equivalent to 10^{-7} Joule). In a similar way, we can also define the *spectral flux density* or *spectral irradiance* as

$$F_\nu = \frac{dE}{dAdtd\nu} = F_E \quad , \quad [F_\nu] = \frac{\text{erg}}{\text{cm}^2\,\text{s}\,\text{Hz}}. \tag{5.1.2}$$

It can be defined as Flux per unit of frequency, that is the energy per unit area, per unit time, per unit frequency. Often a dependency on frequency is denoted by the term "specific", i.e. *specific flux*, meaning nothing more than "how much flux in which energy band?". Note also that one could define the spectral irradiance per unit of energy instead of frequency.

[1] In case you want to refresh your memory, just take a look at Appendix

Total flux

The total flux in an energy (or frequency!) band can be determined by integrating the flux in dE by the limits of the energy range:

$$F_{E2-E1} = \int_{E_1}^{E_2} F_E(eVcm^{-2}s^{-1}eV^{-1})dE \tag{5.1.3}$$

Some useful relations to convert total fluxes are:

$$F_\nu(\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}) = 3.336\cdot 10^{-19}\lambda^2(\text{Å})\cdot F_\lambda(\mathrm{erg\,cm^{-2}\,s^{-1}}\text{Å}) \tag{5.1.4}$$

$$F_\nu(\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}) = 6.626\cdot 10^{-27}F_E(\mathrm{eV\,cm^{-2}\,s^{-1}\,eV^{-1}}) \tag{5.1.5}$$

Intensity

The specific intensity is defined as energy per unit area per unit time per unit frequency (or equivalently per unit of energy) per unit solid angle ("Energy per unit everything"). Note here that I_ν does not depend on θ as it is a feature of the source, whereas the energy recorded at the observer dE does depend on θ.

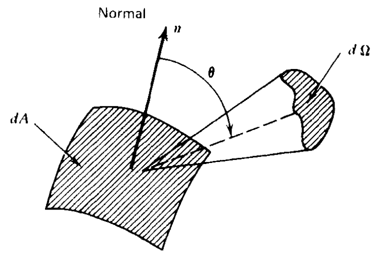


Figure 5.1.2: Geometry of an obliquely incident ray

we have

$$dE_\nu = I_\nu \cos\theta dA dt d\nu d\Omega \quad , \quad [I_\nu] = \frac{\mathrm{erg}}{\mathrm{cm^2\,s\,Hz\,Ster}}. \tag{5.1.6}$$

The solid angle $d\Omega$ is a measure of the amount of the field of view from some particular point. It is given as

$$d\Omega = \frac{A}{r^2} = \sin\theta d\theta d\phi, \tag{5.1.7}$$

where A denotes the spherical surface area and r is the radius of the considered sphere.
Note:

- Stars are *point sources*
- Sun, Moon, Supernova etc. are *diffuse sources*

The intensity is independent of the source's distance: it is an intrinsic property of the spurce.
If we take an integration of Intensity over all solid angles we will obtain the total flux!

From Intensity to Flux

Integrating intensity in all directions would give us the power per unit area and frequency received from the source, which we happen to be known by the name of spectral flux density, or spectral irradiance F_ν:

$$F_\nu = \int_\Omega I_\nu \cos\theta d\Omega \tag{5.1.8}$$

In the case of an isotropic field, we can calculate that $F_\nu = 0$ since I_ν is isotropic (const.) and $\int cos\theta d\Omega = 0$. An example is given by the Cosmic Microwave Background (*CMB*).
The total flux can be obtained by integrating in all frequencies:

$$F = \int_0^\infty F_\nu d\nu \tag{5.1.9}$$

This property again depends on the distance, as $dA = 4\pi R^2$.

Energy density

In a given volume, the amount of energy per unit frequency per unit of solid angle and per volume is defined as the specific energy density u_ν.

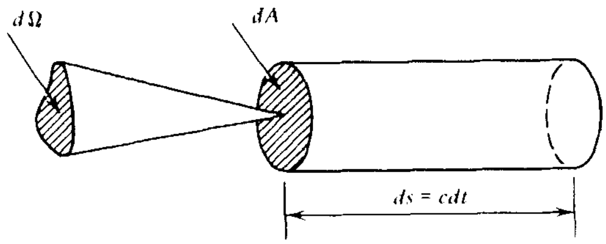


Figure 5.1.3: Energy in cylindrical element

$$u_\nu = \frac{dE}{dV d\nu d\Omega} \qquad dE = u_\nu dV d\nu d\Omega \tag{5.1.10}$$

We know that: $dV = dAcdt$, since photons moves with the speed of light c, and relativistic particle with velocities close to it. Together with

$$dE = I_\nu dA d\nu d\Omega \tag{5.1.11}$$

we get that: $u_\nu = \frac{I_\nu}{c}$ by comparing both expressions for dE. If we integrate over the solid angle $d\Omega$ we obtain

$$U_\nu = \int \frac{I_\nu}{c} d\Omega, \tag{5.1.12}$$

which is the *total spectral energy density*.
Extra information that may be useful:

- Specific radiation density: $J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega \rightarrow$ normalized over space
- $u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c} \int I_\nu d\Omega = \frac{4\pi}{c} J_\nu$

Momentum Flux Density

The momentum per unit of time dt, per unit frequency of $d\nu$, per unit of area dA, from radiation arriving from the solid angle $d\Omega$ is given by the expression

$$dp_\nu = \frac{dE_\nu}{c} \quad = \quad \frac{I_\nu}{c}\cos\theta dA dt d\nu d\Omega. \tag{5.1.13}$$

For the component orthogonal to the area we get

$$dp_\nu \cos\theta = \frac{dE_\nu}{c} \quad = \quad \frac{I_\nu}{c}\cos^2\theta dA dt d_\nu d_\Omega. \tag{5.1.14}$$

For an isotropic field: Flux is zero, but momentum flux density is <u>*not*</u> *zero since it is proportional to* $\cos^2$.
The momentum flux density is then given as

$$\Pi_\nu = \int_\Omega \frac{I_\nu}{c} cos^2\theta d\Omega = \int_\Omega P_\nu \cdot \cos(\theta) d\Omega \tag{5.1.15}$$

Integrating Π_ν over $d\nu$, we obtain the momentum per time and per unit area, which is nothing else than the force per unit area. To wrap one's head around this, it is useful to remember from classical mechanics that

- Momentum per unit time is force.
- Force per unit area is pressure.

The momentum per unit time dt, per unit frequency $d\nu$, per unit area dA, along all directions is given by:

$$P = \int_\nu \Pi_\nu d\nu = \int_\nu \int_\Omega d\nu \frac{I_\nu}{c} cos^2\theta d\Omega \tag{5.1.16}$$

$$\Pi_\nu = \frac{4\pi I_\nu}{3c} = \frac{U_\nu}{3} \tag{5.1.17}$$

$$U_\nu = \frac{4\pi I_\nu}{c} \quad ; \quad U = \int_\nu \frac{4\pi}{c} I_\nu d\nu \quad \rightarrow \quad P = \frac{U}{3} \tag{5.1.18}$$

The relation that connects energy with pressure is called the***equation of state*** **(EOS)**.

- Proportionality constant of pressure and energy density is $\frac{1}{3}$ which is a linear relation.
- Non-linear relation exists: in white dwarfs with proportionality of $\frac{4}{3}$

Further details on the EOS will be provided in the chapter on white dwarfs. For the moment it is enough to mention that it exists.

Luminosity

The Luminosity is an *intrinsic property* of a celestial object. Let us assume that in an interval *dt* the total energy *dE* is irradiated by the star is perpendicular to the surface. *Luminosity* is the total energy irradiated per unit of time *(Power)*. The *Specific Luminosity* is then the total energy irradiated per unit of time and frequency

$$L = \frac{dE}{dt} \quad , \quad [L] = \frac{ergs}{s}, \tag{5.1.19}$$

$$L_\nu = \frac{dE}{dt d\nu} \quad , \quad [L_\nu] = \frac{ergs}{sHz}. \tag{5.1.20}$$

Assuming that a star is a spherical, isotropic emitter (independent of θ and ϕ), with the radius R, then on the surface the relation between L and F is given by:

$$L = F \cdot 4\pi r^2 \tag{5.1.21}$$

Note also that, if no energy is lost or acquired during light propagation, we can find the distance to the star using the relation of inverse square law

$$F = \frac{L}{4\pi D^2}, \tag{5.1.22}$$

where D is the distance to the object. *This is important as if you can measure the flux and know the distance you can measure the luminosity, or the distance if you know L.*
Some additional facts:

- The luminosity of the Sun[2] is measured to be $L_\odot = 3.826 \cdot 10^{26} W$
- The flux at Earth's surface that originated from the Sun is named "solar constant" and is determined to be $S = 1.36 \frac{kW}{m^2}$

.

Important (Exam)

Calculate Flux of Sun-like star is given at a distance d. Alternatively, the flux at the surface is given if you know the radius.

5.2 Radiation transport

Intensity of the radiation can change during propagation due to absorption of particles of the beam or to emission into the beam. In other words, when the beam traverses the medium, energy (let's focus on photons here) can be produced or it can be removed by absorption (photons disappear) or scattering (photons are moved out of line of sight). Hence both, *emissivity* as well as *absorption*, play an important role in the radiation transport.

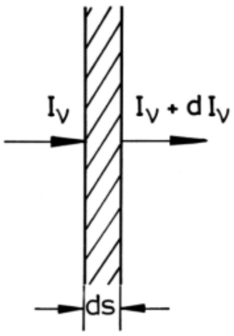


Figure 5.2.1: Attenuation of radiation

The **emissivity** is defined as the energy radiated from a volume dV per unit of time dt and unit of frequency dν in the elementary solid angle dΩ. In general, the emissivity may depend on the **direction**. For example, you could think of the direction of the B-field in the case of synchrotron

[2]These constants, along with many others and additional useful unit conversions can be found in the Appendix

radiation or of jets. The emissivity is independent of the direction in the **isotropic case**. Isotropic emissivity is due to emitters emitting isotropically (such as atoms, molecules, particles, etc.), or due to a superposition of randomly distributed emitters.

We can express the Intensity I_ν that varies over a certain distance ds as it follows:

$$\frac{dI_\nu}{ds} = J_\nu - \alpha_\nu I_\nu \tag{5.2.1}$$

where:

- ds = element of the medium
- I_ν = Intensity variation in the ds element
- J_ν = Emissivity
- α_ν = Absorption coefficient

Equation 5.2.1 is an inhomogeneous differential equation of first order and can be solved with standard and numerical methods. In the following we will look at two special cases that simplify the solution.

Absorption only

When there is only absorption, we have $J_\nu = 0, \alpha_\nu \neq 0$ and equation 5.2.1 becomes

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu \tag{5.2.2}$$

$$\Leftrightarrow \frac{dI_\nu}{I_\nu} = -\alpha_\nu ds. \tag{5.2.3}$$

Integrating the left-hand side yields

$$\ln(I_\nu) = -\alpha_\nu ds \tag{5.2.4}$$

$$\rightarrow I_\nu(s) = I_0 e^{-\alpha_\nu ds}. \tag{5.2.5}$$

By defining

$$\alpha_\nu ds \rightarrow d\tau_\nu(s) = \alpha_\nu ds, \tag{5.2.6}$$

where τ is defined as the *optical depth*. We finally obtain:

$$I_\nu(s) = I_0 e^{-\tau(s)}, \tag{5.2.7}$$

which may already be familiar to you as the BEER–LAMBERT law. In words the above law tells us, that the intensity after the beam has travelled a distance s, is the old intensity times 1/e for $\tau = 1$.
Optically thick medium $\tau >> 1$: low interaction probability.
Optically thin medium $\tau << 1$: high interaction probability.
Note that the optical depth τ depends on the properties of medium and its size.

Important (Exam)

What does radiation transport mean and how is the intensity modified by emission and absorption? Which is the equation of radiation transport?

[2]The concept of Bremsstrahlung is repeated in the Appendix.

Emission only

Contrary to before, in the case of emission only we have $J_\nu \neq 0; \alpha_\nu = 0$. Thus, the equation 5.2.1 takes the form

$$\frac{dI_\nu}{ds} = J_\nu. \tag{5.2.8}$$

If the emissivity J is assumed to be constant, then

$$I_\nu = I_\nu(0) + J_\nu(s)s. \tag{5.2.9}$$

If the general radiation transport equation is divided by α_ν we get

$$\frac{dI_\nu}{ds\alpha_\nu} = \frac{J_\nu}{\alpha_\nu} - I_\nu. \tag{5.2.10}$$

By defining $S_\nu = \frac{J_\nu}{\alpha_\nu}$, the above relation becomes

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu. \tag{5.2.11}$$

We consider two cases:

- If $I_\nu < S_\nu \rightarrow \frac{dI_\nu}{d\tau_\nu} > 0$: Specific Intensity I_ν is increasing towards the source function S_ν
- If $I_\nu > S_\nu \rightarrow \frac{dI_\nu}{d\tau_\nu} < 0$: Specific Intensity I_ν is decreasing towards the source function S_ν

If the system is in thermal equilibrium, the specific intensity doesn't change: as much radiation is emitted as is being absorbed. The solution in this case is given by the Planck spectrum, also known as the black body spectrum. **Black body radiation is radiation which is itself in thermal equilibrium**.

5.2.1 Black body Radiation

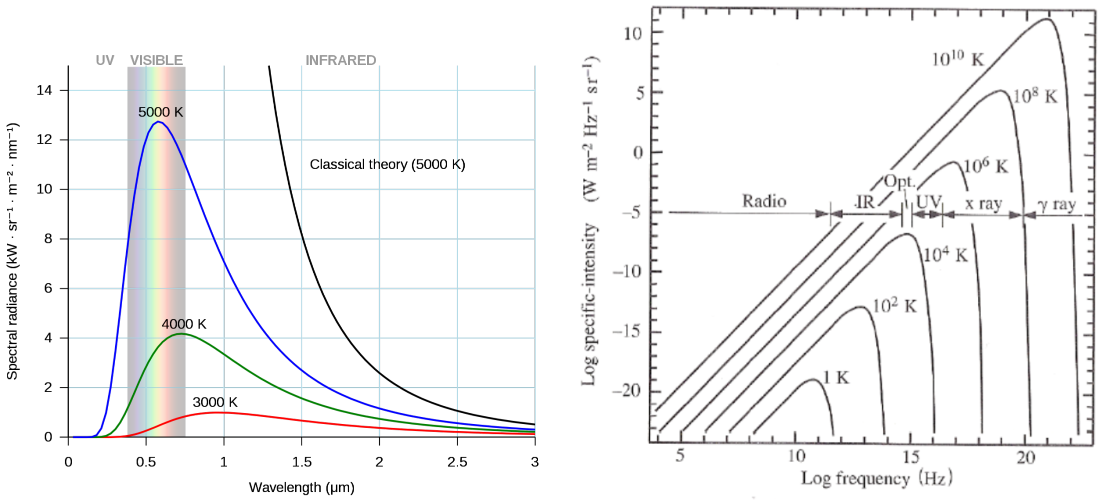
In chapter 3 we used the concept of a *black body* (from now on BB). A BB is an idealized object, in a given thermal state characterized by a given temperature, in equilibrium. We can say that a BB emits and absorbs electromagnetic radiation of all frequencies so that the temperature does not change. The specific intensity observed by this object is the *BB radiation*. The name 'black body' comes from the fact that it absorbs all incident EM radiation, but a BB is not necessarily black (see e.g. the Sun, which can be approximated by a BB).
To put it in a nutshell, a BB is a body which absorbs perfectly and emits BB radiation.
In equilibrium, emission equals absorption so that $S_\nu = I_\nu = B_\nu$ which is the so-called *Plank function*. We will not derive the BB spectral intensity here. The derivation is based on **two assumptions**. First, we assume that photons are **bosons**. Therefore, more than one boson can occupy each quantum state. Second, the photons are in **thermodynamic equilibrium** at all frequencies. Therefore we want to know how many states ϱ_{states} per unit of volume, frequency, and solid angle are possible. Then we calculate how many photons can stay in each state and what their energy is. Eventually, we multiply this by the density of states and get the energy per unit volume, frequency, and solid angle, i.e. the specific energy density μ_ν. A more detailed explanation of this topic can be found in the lecture notes of the module 'High Energy Astrophysics'.
The Planck spectrum is given as:

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \quad ; \quad [B_\nu(T)] = \frac{ergs}{scm^2Hzsr} \quad \rightarrow \frac{W}{m^2Hzsr} \tag{5.2.12}$$

And as a function of wavelength we find

$$B_\nu(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}, \tag{5.2.13}$$

with $\lambda = \frac{c}{\nu}$ being the wavelength.



(a) BBR as a function of wavelength

(b) BBR as a function of frequency

Figure 5.2.2: The black body spectrum

As temperature increases, *Intensity increases* as well as the energy of the photons. For example, in the case of 1 keV we get:

- $\nu = 2.4 \cdot 10^{17}$ Hz
- T = 11.6 $\cdot 10^{6}$ K

As you can see in the figure above, changing the temperature strongly changes the intensity. The peak is shifted toward higher energies. This tells you that the higher the temperature, the higher the frequency (energy!) of the photons in the radiation field. The area under the curve increases. This means that the number of photons populating each state ν increases and photons of higher energies are created, or if you prefer, higher frequency states are occupied.

Important (Exam)

What are the main features of the black body spectrum?

The Rayleigh-Jean and Wien's laws are limits of Planck's function from equation 5.2.13. In the following we mention their main features.

Rayleigh-Jean Approximation

The Rayleigh-Jeans law is obtained for $h\nu \ll kT$, that is for low frequencies:

$$\boxed{I_\nu^{h\nu \ll kT} = \frac{2\nu^2}{c^2} kT} \tag{5.2.14}$$

It is a classical result, this equation approximates the ascending part of the spectrum very well. As you can see, the Planck constant h does not appear in the spectrum anymore, confirming that at low energies quantum effects can be neglected. However, if we want to calculate the total energy Rayleigh Jean approximation leads to the so called 'UV catastrophe'. With 'UV catastrophe' we mean that the intensity diverges when integrated over all frequencies.

Wien's law

For higher energies ($h\nu >> kT$): Using Planck's constant for quantised photons from before we arrive at

$$I_\nu = \frac{2h\nu^3}{c^2} exp\left(\frac{-h\nu}{kT}\right). \tag{5.2.15}$$

It's the description of the specific intensity of the BB radiation at the higher frequencies. It is a result of quantum theory, and h appears in the function. The spectrum drops rapidly after reaching the maximum, because of the exponential roll off. We therefore avoid the ultraviolet catastrophe and include the contribution to the flux of the highest energies.

Wien's displacement law

Wien's displacement law states that the radiation curve of a black body for different temperatures will have its maximum at different wavelengths, which are inversely proportional to the temperature T. That is,

$$\frac{\partial B_\nu}{\partial \nu} = 0 \rightarrow h\nu_{max} = 2.82kT \tag{5.2.16}$$

$$\frac{\partial B_\lambda}{\partial \lambda} = 0 \rightarrow \lambda_{max} T = 0.29cmK. \tag{5.2.17}$$

The temperature of a star is measured using the spectrum and flux which depend on the temperature. By integrating $B_\nu(T)$ over the solid angle for all frequencies, under the assumption that the body is a spherical emitter, we can find the relation between flux and effective temperature: *Stefan-Boltzmann Equation*

$$B(T) = \int_0^\infty B_\nu(T) d\nu = \frac{1}{\pi}\sigma_{SB} T_{eff}^4 \tag{5.2.18}$$

with

$$\sigma_{SB} = \frac{2\pi^5 k^4}{15c^2h^3} = 5.671 \cdot 10^{-5} ergscm^{-2}K^{-4}s \tag{5.2.19}$$

And the flux takes the form

$$F(T) = \int_0^{2\pi} Bcos(\theta) d\Omega = B\pi = \sigma_{SB} T_{eff}^4. \tag{5.2.20}$$

The Stefan-Boltzmann constant σ_{SB} is a result of quantum physics, since it includes the Planck's constant h. The flux is proportional to the fourth power of the temperature. Note the we have introduced a new symbol: T_{eff}. This stands for 'effective Temperature'. We obtain the effective temperature by equating the flux F to that of a BB at the same temperature. The luminosity can also be calculated for a spherically symmetric emitter using

$$L = 4\pi R^2 F = 4\pi R^2 \sigma_{SB} T_{eff}^4, \tag{5.2.21}$$

from which we can derive the actual radius of the star

$$R = \sqrt{\frac{L}{4\pi\sigma_{SB} T_{eff}^4}}. \tag{5.2.22}$$

In words: If we have the flux and luminosity we can find the *radius* of an object. *(very important result for radius of the white dwarfs! Because of low luminosity, but high temperatures it leads to low radius)*

5.3 Magnitudes and Colours

5.3.1 Magnitudes

What is the magnitude of a star, or more in general a celestial object? In the following, we will discuss the different types of magnitudes, as well as some other concepts connected to the magnitude. Let's say that the magnitude is a parameter related to the brightness, apparent or absolute, of a celestial object. The Greek astronomers Hipparchus (190-120 BC) and Ptolemy (170-100 BC) were the first to compile comprehensive star catalogs in the western world. In the catalog, they included positions and brightness of the stars. The units of brightness are defined 'magnitudes', m, expressed in mag. The scale of magnitudes goes from $m = 1$ for the brightest stars to $m = 6$ for the faintest.

Apparent Magnitude

The scale uses an 'apparent' magnitude, which depends on the star's brightness and distance. The apparent magnitude m is the magnitude that we can see/measure on Earth. It follows a logarithmic scale. Bright stars have a low magnitude. Faint star have a high magnitude (faintest (m= 30 observed by HST), human eye lower limit = 6 mag). In modern times (1850) Pogson suggested a scale in which a difference of 5 magnitudes corresponds exactly to a factor of 100 in flux (or equally said in brightness). The change in one order of mag then corresponds to a factor of

$$2.512 = 100^{1/5}. \tag{5.3.1}$$

A star with two orders of magnitude = $100^{2/5}$ or 2.512^2 with

$$m_1 - m_2 = -2.5log_{10}\left(\frac{F_1}{F_2}\right) \tag{5.3.2}$$

It can also be used as

$$\frac{F_1}{F_2} = 10^{-0.4(m_1-m_2)} \tag{5.3.3}$$

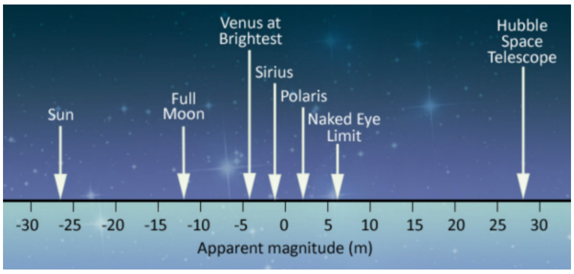


Figure 5.3.1: Apparent magnitudes scale of objects in the sky.

Often we consider the apparent magnitude at a defined wavelength (or frequency). This can be expressed as:

$$m_\lambda = -2.5log_{10}\frac{F_\lambda}{F_{\lambda 0}} = -2.5log_{10}[\int_0^{+\infty} F_\lambda^* S_\lambda d\lambda] + c_\lambda \tag{5.3.4}$$

Object	Apparent magnitude(mean values in visual)
Sun	-27
Full moon	-13
Venus	-4.1
Jupiter	-2.2
Sirius	-1.5 (brightest star in the night sky)
Vega	+0.03(definition of zero magnitude)
Saturn	+0.5
Mars	+0.7
Mercury	+0.2
Andromeda/M31	+3.4
Ganymede	+4.4(at best)
M33	+5.7(used as eye check)
Uranus	+5.7
Neptune	+7.8

Table 5.2: Examples of apparent magnitudes of astronomical objects

Absolute magnitude

The 'absolute' magnitude is denoted with the capital letter M. This is the apparent magnitude a star would have if it were located at a distance of 10 pc - normalised apparent magnitude. Thus, the absolute magnitude is independent from the distance to the source.

$$100^{(m-M)/5} = \frac{F_{10pc}}{F} = (\frac{d}{10pc})^2 \tag{5.3.5}$$

The quantity **(m -M)** is called **distance modulus** and is defined as

$$m - M = 5log_{10}(\frac{d}{10pc}). \tag{5.3.6}$$

In the case of the sun, the absolute magnitude is much smaller than the apparent magnitude, because at 10pc the flux from the sun would be much smaller (since the Sun is located at 1 AU =~ 4.85^{-6}pc).

$$100^{(M_1-M_2)/5} = \frac{L_1}{L_2} \tag{5.3.7}$$

For two stars at the same distance the ratio between their fluxes is equal to the ratio between their luminosities. The ratio between absolute magnitudes, corresponds to the ratio of the luminosities, since they are at same distance of 10 pc.*For the pulsating variables stars, the luminosity can be determined without the knowledge of their distances, and from the known luminosity we can always find their distance.*

Important (Exam)

What are apparent and absolute magnitude? How are these quantities changed by extinction?

True magnitude

The true magnitude is is a measurement of the star's luminosity. However, light can be absorbed along its propagation path to Earth. In fact, space between the source and the observer is in general not empty and contains the interstellar medium (ISM). We might have obscuration due to the ISM.

There will be extinction of the radiation due to ISM absorption along the line of sight. This is then called *Interstellar extinction*. Interstellar extinction changes the apparent magnitude of the star and hence the distance modulus which must be corrected for.

$$m_\lambda = M_\lambda + 5\log_{10}(d) - 5 + A_\lambda, \tag{5.3.8}$$

where $A_\lambda = 1,086T_v$ is the interstellar extinction.

5.3.2 UBVRI Photometric System

A photometric system is a passband with sensitivity to the incident radiation. Sensitivity depends on the filters used during the detection.

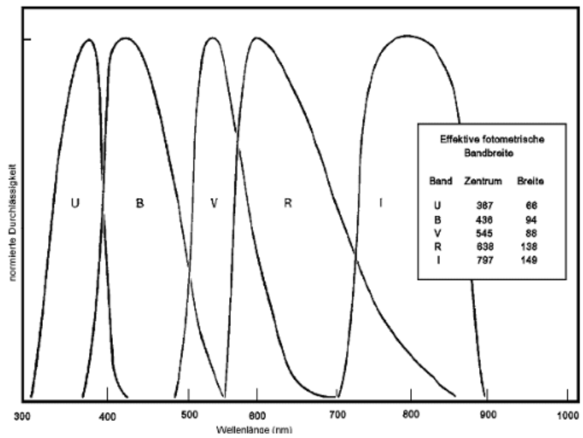


Figure 5.3.2: The UBVRI bands

Filter	Color	λ_{eff} [nm]	$\Delta\lambda$ [nm]
U	UV	365	66
B	Blue	445	94
V	Visual	551	88
R	Red	658	138
I	IR	806	149

Table 5.3: Passbands for the UBVRI filters

Colour Index(C.I.)

The C.I. relates to the difference between two absolute magnitudes.

$$C.I = M_X - M_Y \qquad X, Y : UBVRI \tag{5.3.9}$$

X denotes the smaller wavelength while Y denotes the larger wavelength. The C.I. is related to the *photospheric temperature* of the star (Planck's Spectrum)
For example:

$$U - V = M_U - M_V \tag{5.3.10}$$

Colour index gives an idea of the energy distribution of the star. As the brightness increases, magnitude decreases hence the star with smaller B-V will be bluer than the star with larger B-V.

Bolometric Correction(B.C)

Apparent and absolute magnitudes measured over ALL wavelengths from the star are defined as bolometric magnitudes. They are then usually denoted as absolute bolometric magnitude (M_{bol}) and Apparent bolometric magnitude (m_{bol}).
The bolometric correction (B.C.) is the difference between the star's apparent bolometric magnitude and its visual magnitude

$$B.C. = m_{bol} - V = M_{bol} - M_V \qquad (V = m_V). \tag{5.3.11}$$

We can take the similar expression in all the bands using the relation between Flux and magnitudes.

$$\begin{aligned} m_\lambda &= -2.5\log_{10}\left(\int_0^{+\infty} F_\lambda S_\lambda d\lambda\right) + C_\lambda \\ U &= -2.5\log_{10}\left(\int_0^{+\infty} F_\lambda S_U d\lambda\right) + C_U \end{aligned} \tag{5.3.12}$$

S is the sensitivity and C is a constant which depends on the chosen filters.
Bolometeric brightness:

$$m_{bol} = -2.5\log_{10}\left(\int_0^{+\infty} F_\lambda d\lambda\right) + C_{bol} \tag{5.3.13}$$

'C' was added so that the BC is negative for all stars. However, a few of the supergiants have positive BC but C remains unaltered. 'C' can be calculated from the Sun's magnitude: $m_{sun} = -26.83$.

Color excess

ISM alters the radiation due to various processes such as absorption, scattering or emissions which is called extinction. In particular, the **reddening of the light** is due to scattering and absorption by the ISM. Blue light is scattered more than red light which leads to reddening. Thus, the C.I (B-V) increases.

$$\begin{aligned} V &= M_V + 5\log_{10}\left(\frac{d}{10pc}\right) + A_V \\ B &= M_B + 5\log_{10}\left(\frac{d}{10pc}\right) + A_B \end{aligned} \tag{5.3.14}$$

Observed C.I :

$$\begin{aligned} B - V &= M_B - M_V + A_B - A_V \\ B - V &= (B - V)_0 + E_{B-V} \end{aligned} \tag{5.3.15}$$

E_{B-V} is the color excess, that is, the difference between intrinsic C.I and observed C.I while $(B-V)_0$ *is an intrinsic color*.
Extinction law:

$$A_V = 3.1 \cdot E_{B-V} \tag{5.3.16}$$

Important (Exam)

What are colours in astronomy?

Some belated changes from the International Astronomical Union (IAU)

- Resolution B2 defines: $M_{bol} = 0$ corresponds to $L_0 = 3.012 \cdot 10^{28} W$
  L_0 is the zero point luminosity, such that, $L_{bol,\odot}$ corresponds to $M_{bol,\odot} = 4.74$
- Total solar irradiance measured at 1pc ($1361 W m^{-2}$) corresponds to $m_{bol,\odot} = -26.832$
- Following the above point bolometric magnitude of a star is redefined as:$M_{bol} = -2.5 log_{10} \frac{L_\star}{L_0}$
- Luminosity in Watts: $L_\star = L_0 \cdot 10^{-0.4 M_{bol}}$

5.4 Spectral types and lines

Learning goals

- Why do we observe absorption and emission lines in a stars' spectra? What is the 'physics' of their origin?
- How are stars classified? Discuss the different classifications (Harvard, Yarkes...)
- What is a Hertzsprung Russel diagram? What is it's meaning?
- How can we measure the radii, masses and magnetic fields of stars?

Using a prism NEWTON obtained the Sun's spectrum in 1666. In 1814 FRAUNHOFER observed dark absorption lines in the Sun's spectrum, labeling the most prominent from A ... L (see table 5.4) but including about 500 other lines.

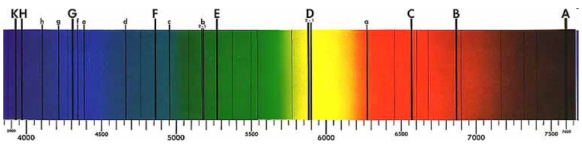


Figure 5.4.1: The spectrum of the Sun

λ(Å)	Name	Element
7593.30	A	O_2
6867.19	B	O_2
6562.18	C	Hα
5895.14	D_1	Na
5889.97	D_2	Na
5183.63	b_1	Mg
5172.70	b_2	Mg
5167.33	b_4	Mg
4810 ± 10	G	CH,Fe
4226.74	g	Ca
4110.75	h	Hδ
3968.49	H	Ca
3933.68	K	Ca
3820.44	L	Fe
3734.87	M	Fe
3581.21	N	Fe

Table 5.4: Important spectral lines of some elements

KIRCHHOFF and BUNSEN identified the Fraunhofer lines as absorption lines due to atomic and molecular transitions in the Sun and Earth's atmosphere. The spectra of stars are not simple black body spectra but are characterized by dark absorption lines or bright emission lines. As we said, the origin of the lines is due to the transitions of electrons between different, quantised energy levels of atoms (or molecules). The centroid energies (or wavelengths) of the lines are a precise feature of the elements in the gas, or in other words, each line identifies an element. Usually, the spectroscopic nomenclature is used:

- I- neutral
- II- 1 fold-ionised
- III- 2 fold-ionised
- etc...

Following the studies by Bunsen and Kirchhoff on lines from elements, the Kirchhoff's laws were established:

- Hot and dense gas/object – continuum spectra, no dark line
- A hot diffused gas – emission lines
- A cool diffused gas in front of source of continuous spectra – dark absorption lines

Examples for a hot gas are the Sun's corona, and the intracluster gas, filling clusters of Galaxies (typically observed in the X-rays). The Sun's photosphere is an example for a cold gas. Interestingly, Helium was discovered spectroscopically in the spectrum of the Sun in 1868 and only later, in 1895, was found on Earth.
It's a result of quantum mechanics that the energy levels of electrons in the atoms are quantised and characterized by quantum numbers. The set-up of which can be seen in figure 5.4.2. As an example, n=1 corresponds to the ground level, while n=2 characterizes the first excited level. Electrons transitions between these levels, that is between bound states, or between a bound state and free state can occur. In the transition a quantum of light, a photon, is emitted or absorbed. More specifically, a photon is emitted in the electron transition from a higher to a lower energy level. On the contrary, a photon is absorbed when the electron makes a transition from a lower to a higher energy level.
Depending on the initial level, we define different so called *series*, like the Lyman, Balmer, Paschen series. The energy difference between bound states is given by ΔE:

$$\Delta E = h\nu = -13.6\left[\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right]. \tag{5.4.1}$$

Where n refers to the energy quantum number of the level. The quantum number n=1 corresponds to the ground level, while n=2 characterizes the first excited level.

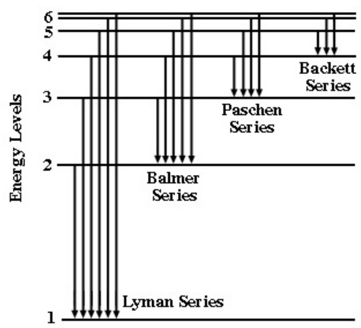


Figure 5.4.2: Series for the transition | *Credit:* Harper College

- Lyman Series: Transition between higher energy level to ground states (n=1)
- Balmer series: Between higher energy level to n=2
- Paschen series: Between higher energy level to n=3
- Energies: Lyman>Balmer> Paschen
- Lyman(UV); Balmer(Visible); Paschen(Infrared)

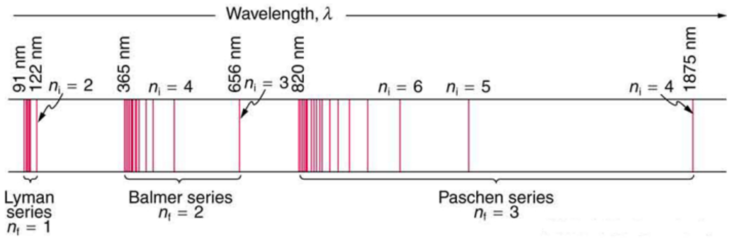


Figure 5.4.3: Transition lines.

Usually there are 4 possible transition types of the electron in the atom:

- Bound-Bound: jump within bound states
- Bound-Free: jump from a bound to a free state
- Free-Bound: jump from a free to a bound state
- Free-Free: scattering from a free to another free state

Observed positions of lines in the spectrum can change due to ***doppler effects, cosmological redshift, gravitational redshifts****, but patterns remain same so we can still use the spectral classification.*

5.4.1 Spectral classification

To give you an idea of the complexity of spectra the figure below shows the spectra of the Sun and of the star Vega

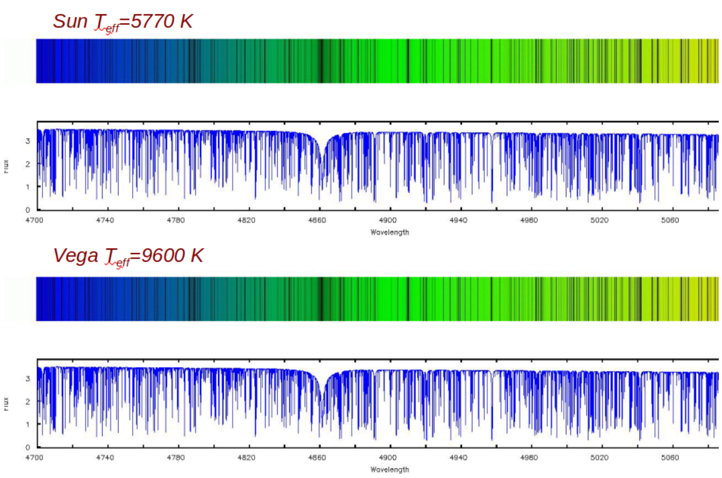


Figure 5.4.4: Spectra of the Sun(top) and the star Vega(bottom)

The spectral types of the stars are defined through the appearance of the spectral lines.

Harvard Classification

This is a taxonomy developed at Harvard, according to the strength of the Hydrogen line absorption. The sequence follows the temperature of the star. It reads as capitalised **O B A F G K M**, with O being the hottest stars, M the coldest.

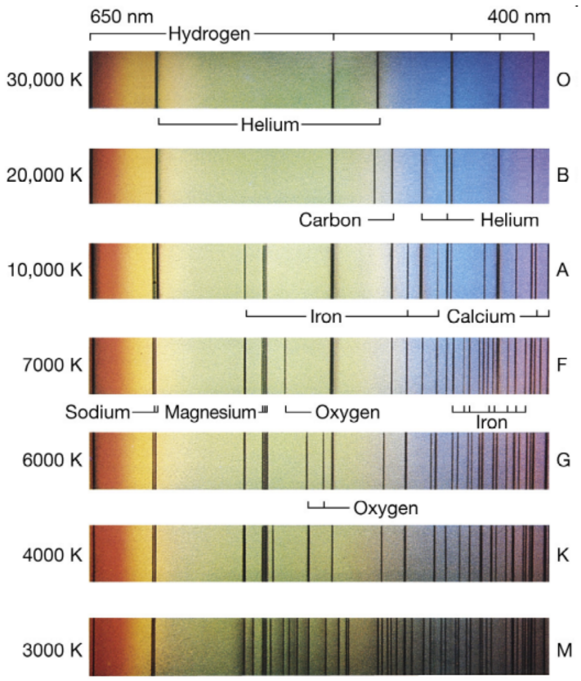


Figure 5.4.5: Spectra for O,B,A,F,G,K,M type stars

Spectral type	Temperature [K]	Description
O	30,000 - 60,000	Hottest blue-white, ionised Atoms, strong HeII
B	10,000 - 30,000	Hot blue-white, neutral He, weak HI
A	7,500 - 10,000	White, H lines strongest, some ionised metals
F	6,000 - 7,500	Yellow-white H and ionised metals
G	5,000 - 6,000	Yellow, very strong Ca II, neutral and ionised metals
K	3,500 - 5,000	Cool Orange neutral metals
M	3,500	Cool Red Strong Molecular lines (TiO)

Table 5.5: Spectral types of stars - Harvard classification

It is defined as starting from hot stars to cold stars. O and B are *early type* stars; whereas, K and M are said to be the *late type* stars. As a rule of thumb, one can say that H lines mean a hotter star, whereas Na lines imply a cooler source. O stars are hottest stars, they are blue and have a great fraction of ionised atoms, with strong He II lines. As we move to lower temperature, and in the spectra towards the M types, more different elements are seen, such as oxygen, sodium etc. Note that:

- In astronomy, everything heavier (more than 2 protons) than He is called a metal
- Subdivisions added in spectral classification ranging from 0 to 9, e.g. A0 to A9.

- L and T types are added late for super dim stars.
- L type have low temperatures, dark red with strong infrared emission and with molecular absorption band.
- T types (Brown dwarf) are the coolest, infrared emission with strong methane band

"Hot and Cooler relative to Sun".

Morgan Keenan (MK- Classification)

This is a luminosity class, which is written in Roman numeral and is linked together with Harvard classification.

Luminosity class	Description	Examples
0 / Ia+	Hypergaints/extremely luminous Supergaints	Cygnus OB2 - B3-4Ia+
Ia	Luminous supergaints	Eta Canis Majoris - B5Ia
Iab	Intermediate-size luminous supergaints	Gamma Cygni - F8Iab
Ib	Less luminous supergaints	Zeta Persei - B1Ib
II	Bright gaints	Beta Leporis - G0II
III	Normal Gaints	Arcturus - K0III
IV	Subgaints	Gamma Cassiopeiae - B0.5IVpe
V	Main-sequence stars(dwarfs)	Achernar - B6Vep
sd or VI	Subdwarfs	HD 149382 - B5VI
D or VII	White dwarfs	van Maanen - DZ8

Table 5.6: Morgan-Keenan classification

White Dwarfs: DA, DB, DZ etc D- (degenerate)
Wolf-Rayet stars: WN, WC etc.

5.4.2 Hertzsprung–Russell diagram

In 1905 the Danish engineer Ejnar Hertzsprung (1873-1967) analyzed stars whose absolute magnitudes and spectral types were accurately known. Hertzsprung published a paper reporting the correlation between these two quantities: Magnitude and spectral types. A similar diagram was independently developed by the U.S. astronomer Henry Norris Russell, which also included (and named) the brightest stars as giants.

The Hertzsprung–Russell diagram (HRD) is a scatter plot of fundamental importance. It shows the relation between absolute magnitude or luminosity and temperature or spectral types. To understand why the diagram is so important, remember from equation 5.2.22, which links temperature and luminosity and can be used to find the radius of the object

$$R = \frac{1}{T_{\text{eff}}^2}\sqrt{\frac{L}{4\pi\sigma}}. \tag{5.4.2}$$

The HR diagram is shown in the figure below.

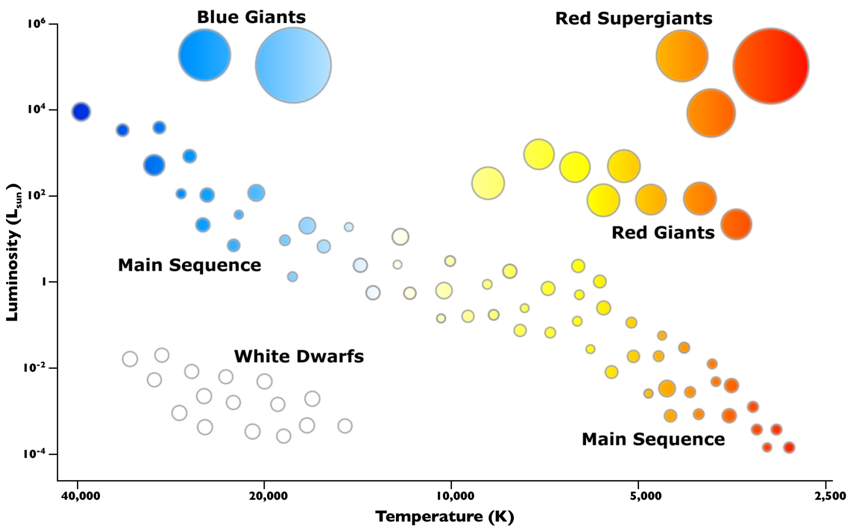


Figure 5.4.6: Hertzsprung–Russell diagram | *Credit:* Mason m · Stefan V.

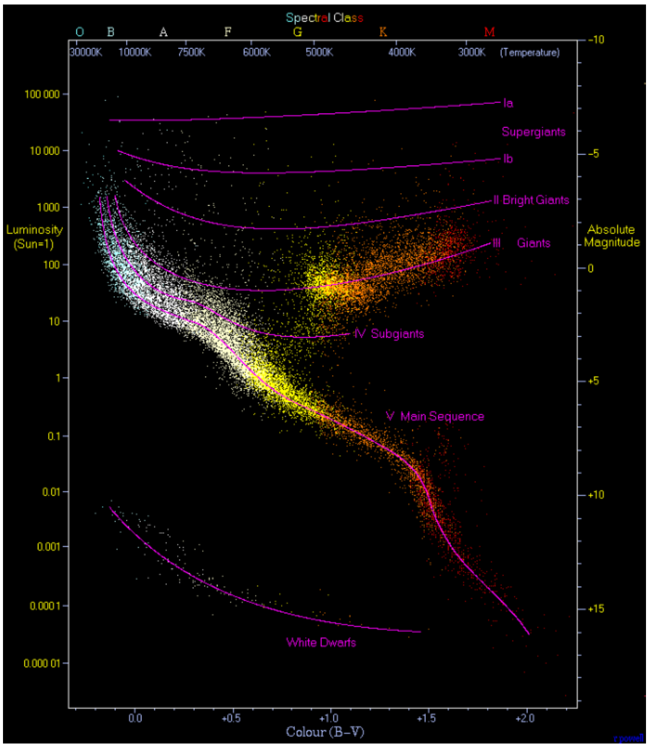


Figure 5.4.7: Hertzsprung–Russell diagram with (B-V) on the x-axis and Luminosity on the y-axis.

The x axis of the HR diagram usually displays the spectral classification of the stars from O to M. Using a more physical quantity, the temperature, or better the effective temperature Teff, decreasing from right to left, is also often shown. On the y axis we plot the absolute magnitude of the star, or the luminosity, typically expressed in solar luminosity units. Note that both Teff and L are plotted on a logarithmic scale. The brightest, hottest stars can reach luminosity values up to $10^6 L_{\odot}$and temperatures up to more than 40,000K. As we will see in the next chapters, the HR diagram is fundamental to trace stellar evolution. In the HR diagram we observe different branches:

- Main sequence: from B to M types, lie diagonally in the HR diagram
- Giants: divided in giants and super giants, they are found in multiple evolution branches above main sequence
- WDs: they are found in a branch below the main sequence

The main sequence contains roughly 95% of the detected stars.The giants are located above main sequence, while the white dwarfs can be found below the main sequence. The evolution of a star, i.e. its track in the HR diagram, is determined by its mass. The HR Diagram allows a comparison of the observed phenomenology with predictions of theories about star interiors and evolution.

Note also that in the (log T_{eff}, log L) diagram, parallel lines correspond to a given radius. We have:

Type	Radii
Main sequence	0.1-20 $R_\odot$
Red Giant	10-200 $R_\odot$
Super Giants	>1000 $R_\odot$
White Dwarf	1/100 $R_\odot$

Table 5.7: Radii of different classes of stars

The Mass-Luminosity relationship

The radius of a star can be measured from L and T. It is however very difficult to measure it directly. This can be done with very few stars and only if we have extremely powerful telescopes. As we will see later, we might also use eclipsing binaries.

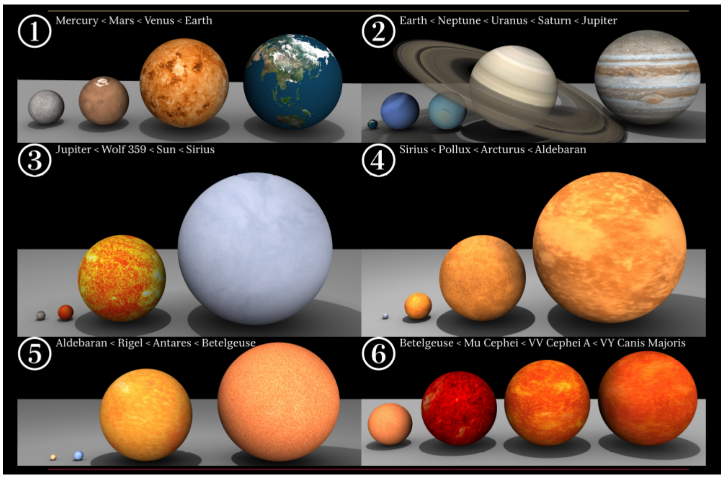


Figure 5.4.8: Size comparison of different stars.

Star	Radius
Sun	6.96×10^{10} cm
Antares	$\approx 500 R_\odot$
Aldebaran	$\approx 40 R_\odot$
Capella	$\approx 15 R_\odot$
Spica	$\approx 7 R_\odot$
Sirius A	$\approx 2 R_\odot$
Jupiter(planet)	$\approx 0.1 R_\odot$
Sirius B	$\approx 0.01 R_\odot$

Table 5.8: Relative radii of stars with respect to the Sun.

We can measure stellar mass using mass-luminosity relation, that is

$$\frac{L_\star}{L_\odot} = a \left(\frac{M_\star}{M_\odot} \right)^b , \tag{5.4.3}$$

where b is called the Eddington limit, a and b depend on the masses of the stars.

Extra points to note:

- We can measure the magnetic field of the star using the Zeeman spectrum of the star.
- Magnetic field of Sun spot: 4000G interior and $\sim$ 1G on the surface
- Zeeman-Doppler imaging: Tomography technique to study periodic change in Zeeman signatures as a function of stellar rotation.

Chapter 6

Binaries

Abstract.

Introduction to the different classes of binaries, their constituents and their visual characteristics. In more detail the different classes will be explained.

Keywords: Binaries, HMXB, LMXB, spectroscopic, eclipse

Learning goals

- How are binaries classified
- What are optical/visual binaries
- What are astrometric/spectroscopic binaries
- Determine the mass of spectroscopic binaries
- What are eclipsing binaries

More than 50% of stars belong to a multi body system orbiting around a common center of mass. Via the orbital parameters one can deduce the other physical properties such as the mass m. E.g. the compact object emission (X-ray) of a body with a mass estimate of $> 3M_\odot$ might be a Black Hole.

6.1 Classification

Binary system are classified based on their observational features. Here is a short overview of the classes of binaries that will be discussed in this chapter.

- **Optical double:** Two stars are very close along the line of sight, however not gravitationally bound. They are not actual binaries.
- **Visual binaries:** Both stars can be resolved independently. If the period isn't too long, the motion of the orbit can be observed. There are approximately 100 000 known. If the angular separation is known and the distance can be calculated, the linear separation can be calculated. The limit for such a observation is 0.1" (terrestrial) and 0.001" (space). -> (See chapter 3.)

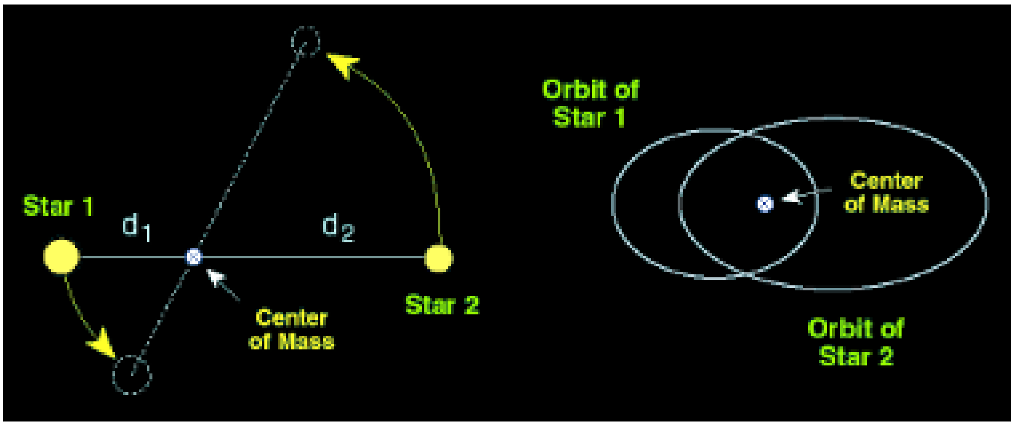


Figure 6.2.1: On the left: distance of star 1 and star 2 to the center of mess. On the right: The orbits of the stars and their center of mass.

- **Astrometric binaries**: If one of the two stars is much brighter than its companion, the companion can't be seen. However it is visible via its gravitational influence. Then main star "oscillates" radially and produces a Doppler shift in its spectra.
- **Ecplipsing binaries:** Inclination is almost 90%. The orbit plane is along the line of sight, which means that one star passes in front of the other. This give an imprint on the light curve.
- **Spectrum binary:** it is a system with two superimposed, independent, discernible spectra. If the stars have a radial velocity component a periodic blue- and red-shift of the lines (Doppler effect). Note that when the lines of one star are red shifted, the lines of the other star will be blue shifted.
- **Spectroscopic binaries:** Only one spectrum is observed. This can be done if the period is short and the radial velocity is along the line of sight a periodic shift of the spectral lines can be observed.

Another way of binary systems is by their distances to each other and their rotaion periode. Wide systems are usually defined with distances between 10 AU up to 1000 AU and periods of 10 to 1000 years. For tight system the distance for the companions can be from 1 AU down to $1R_{\odot}$ with periods from 1 year to a few minutes. The fastest orbital periode observed so far was 5.4 minutes in the system (Which one is it?) that consists of two white dwarfs.

6.2 Visual binaries

Visual binaries can be resolved as independetly and limits of their angular seperation are around 0.1" for earth-bound instruments and down to 0.001" when using intruments in space.
In general, there are two possibly reasons why an elliptic orbit is observed.

a) The eccentricity of the orbit is not zero, $\epsilon \neq 0$
or
b) the orbit is inclinded with respect to our line of sight
If the distance d is known, the observable semiaxis a with angle α is $a = d \cdot sin\alpha$.

When the orbits of the binary system are perpendicular to our line of sight - meaning we have a face on view of the system - the absolut postion of both stars can be measured. Semimajor axis a_1

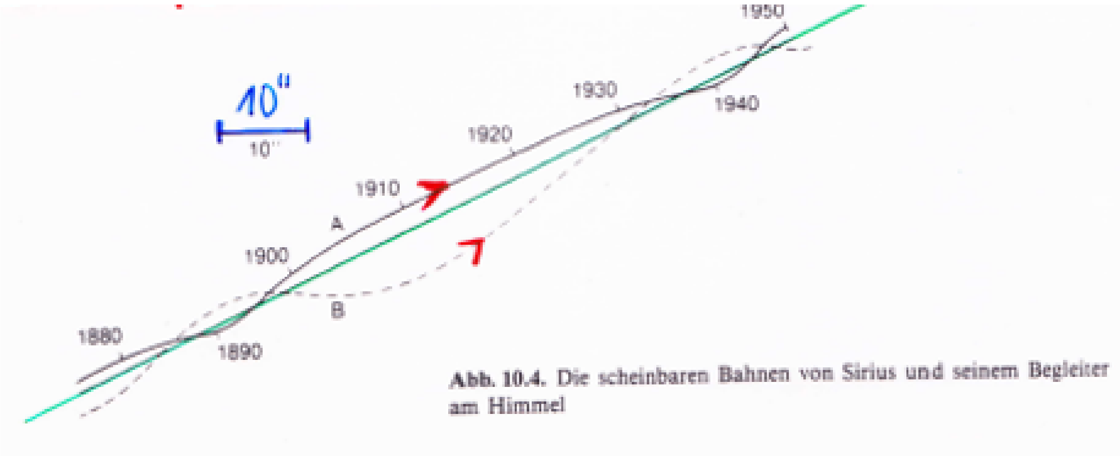


Figure 6.3.1: Movment of Sirius and its companion in night sky.

and a_2 of the two orbits can be determined as follows.

$$a = a_1 + a_2 \tag{6.2.1}$$

is the semimajor axis of the orbit of the reduced mass and center of mass is:

$$m_1 \cdot a_1 = m_2 \cdot a_2 \quad , \quad \rightarrow \frac{m_1}{m_2} = \frac{a_2}{a_1} \tag{6.2.2}$$

If the distance of the binary system is known, then the angles subtended by the semimajor axes are:

$$\alpha_1 \frac{a_1}{d} = \alpha_2 \frac{a_2}{d} \quad , \quad \rightarrow \frac{m_1}{m} = \frac{\alpha_2}{\alpha_1} \tag{6.2.3}$$

which means that even without the distance we can get the mass ratio. Combining the third Kepler law

$$\frac{(a_1 + a_2)^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2} \tag{6.2.4}$$

and Eq. 6.2.3 we can get the total mass if we know the distance.

6.2.1 Inclined orbits

The orbit can be inclined of an angle i with respect to the plan of the sky. The apparent angles are

$$\tilde{\alpha_1} = \frac{a_1 \cdot cos(i)}{d} \quad , \quad \tilde{\alpha_2} = \frac{a_2 \cdot cos(i)}{d} \quad , \quad \frac{m_1}{m_2} = \frac{\tilde{\alpha_2}}{\tilde{\alpha_1}} \tag{6.2.5}$$

In combination with Eq. 6.2.4 we get

$$(m_1 + m_2) = \frac{4\pi^2}{G} \cdot \frac{\tilde{\alpha}^3}{P^2} \cdot (\frac{d}{\cos(i)})^3 \quad , \quad with \quad \tilde{\alpha} = \tilde{\alpha_1} + \tilde{\alpha_2} \tag{6.2.6}$$

However we need to know i, which is often not given.

6.3 Astrometric binaries

A well known example is Sirius with it's white dwarf companion. Figure 6.3.1 shows the paths of Sirius A and its white dwarf companion Sirius B between 1880 and 1950. In Table ?? we can see the compaison of the two stars and see that the Sirius B is a lot dimmer not just than Sirius A but also a lot less luminous than the Sun althought they are about the mass. There are ≈ 20 confirmed astrometric binaries and within 5 pc, 20% of all stars have unseen companions.

Object	Luminosity [$L_\odot$]	Mass [$M_\odot$]
Sirius A	23.5	2.3
Sirius B	0.03	1.053

Table 6.1: Comparison of Sirius A & Sirius B. Almost same mass but far greater luminosity for Sirius A. Sirius B is the first discovered white dwarf which commonly have low L but high T.

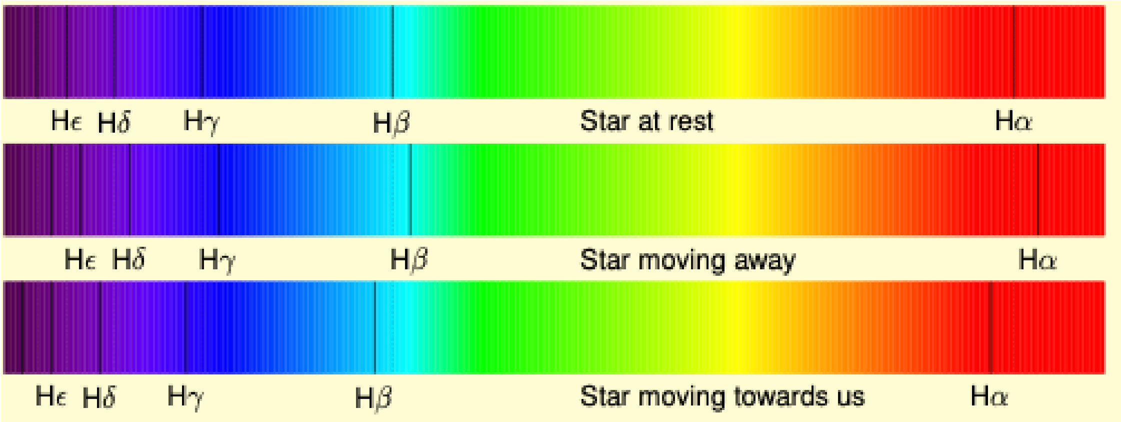


Figure 6.4.1: Shift of Hydrogen lines due to motion of the source

6.4 Spectroscopic Binaries

In spectroscopic binaries shifted absorption lines can be seen in the spectrum like in **??**. The lines are shifted because of the motion of the source. With the difference in wavelength we can calculate the radial velocity of the source (see Equation 6.4.1).

$$\frac{\Delta\lambda}{\lambda} = \frac{v_{\mathrm{rad}}}{c} \tag{6.4.1}$$

Alternative observation: Radial velocity curves for the case of two spectra. K1, K2 are the half velocity amplitudes: these are measured values. They are only sinusoidal if the orbits are circular. Ellipticities induce a change of the function shape. he radial velocity of the two stars, can be determined from the Doppler effect. We can only measure the radial velocity along the line of sight circular orbits:

$$\theta = wt \quad , \quad w = \frac{2\pi}{p} \tag{6.4.2}$$

We observe

$$v_{\mathrm{r}} = v\cos(wt) \tag{6.4.3}$$

and in dependency of i we obtain

$$v_{\mathrm{r}} = v\cos(wt)\sin(i) \tag{6.4.4}$$

$$K_1 = \frac{2\pi a_1}{P}\cdot\sin(i) \quad , \quad K_2 = \frac{2\pi a_2}{P}\cdot\sin(i) \tag{6.4.5}$$

With

$$a_1\sin(i) = \frac{P}{2\pi a_1}\cdot K \tag{6.4.6}$$

we get

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{K_2}{K_1} \tag{6.4.7}$$

Again from Eq. 6.2.6 we get

$$(m_1+m_2)\cdot\sin(i)^3 = \frac{P}{2\pi G}\cdot(K_1+K_2)^3 \tag{6.4.8}$$

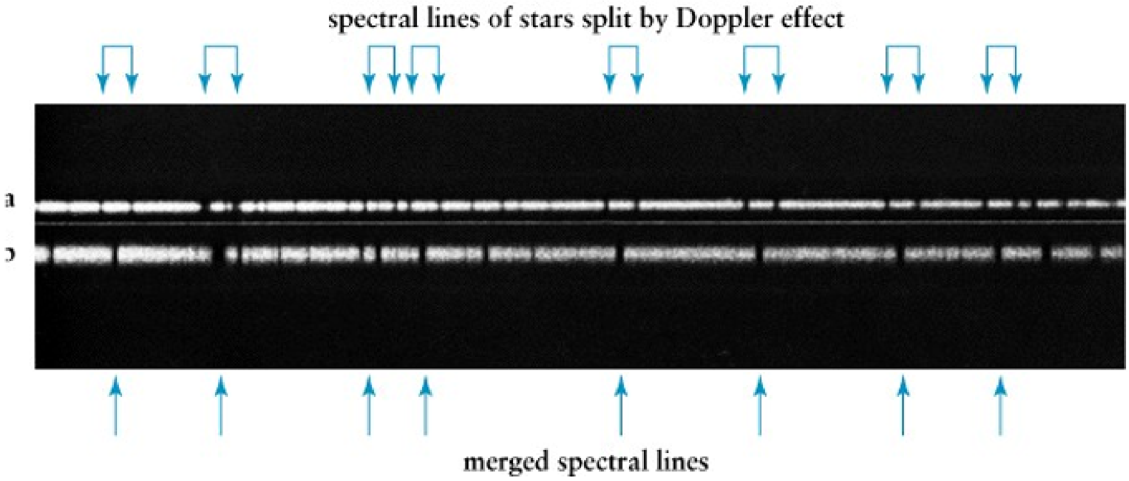


Figure 6.4.2: Spectrum with spilt absorption lines

Again we need to know i. For a single line spectroscopy we are left with

$$K_2 = K_1 \frac{m_1}{m_2} \tag{6.4.9}$$

Combining Eq.6.4.8 and Eq.6.4.9 we get

$$(m_1 + m_2) = \frac{P}{2\pi G} \cdot \frac{K_1{}^3}{sin(i)^3} \cdot (1 + \frac{m_1}{m_2})^3 \tag{6.4.10}$$

This gives us the **Mass equation** which sets a lower limit on the mass for a second body, if we only know one of the two.

$$f(m) = (\frac{m_2}{m_1 + m_2})^2 sin(i)^3 = \frac{P}{2\pi G} K_1{}^3 \tag{6.4.11}$$

The lower limit is given by *f(m)* ≤ m_2. This can be especially useful for estimating the mass of non-emitting stars like a potential Black Hole. Usually $\sin(i)^3$ is averaged at 0.589 or $\frac{2}{3}$ in order to calculate the mass function. Even if both radial velocities are measurable, it is not possible to measure the mass if we don‘t know i, the inclination angle.

6.5 Eclipsing binaries

When binaries pass in front or behind one another, a dip in the luminosity can be detected. In Figure 6.5.1 two types of escipses are shown. When the obsects are similar in size, the eclipse is often patial, that means none of the stars are fully covered by the other. For binaries with different sizes, total coverage is possible. The light curve shows a big dip, if the secondary object passes in front of the primary. A second smaller dip is provided when the secondary object disappears behind the primary.
When a *fourier transformation* is applied to the light curve a power spectrum is attained which will peak at the frequencies which contribute to the variability of the system. Partial or total occultation provides a good estimate of sin i ≈ 1 at i about 90° (even if i would be 75° the error on sin(i) would only be of the 10%). This allows an estimate of the masses when both spectra are observed. An estimate of the radii of the binary members is possible.

6.5.1 Estimate of the radii in eclipsing binaries

If we know the time of the first contact t_1 and of the last contact t_4, and of the minimum light t_2 at the ingress, and at the egress t_3 plus the relative velocity of the stars (from Doppler effect).

$$D + d = v \cdot (t_4 - t_1) \tag{6.5.1}$$

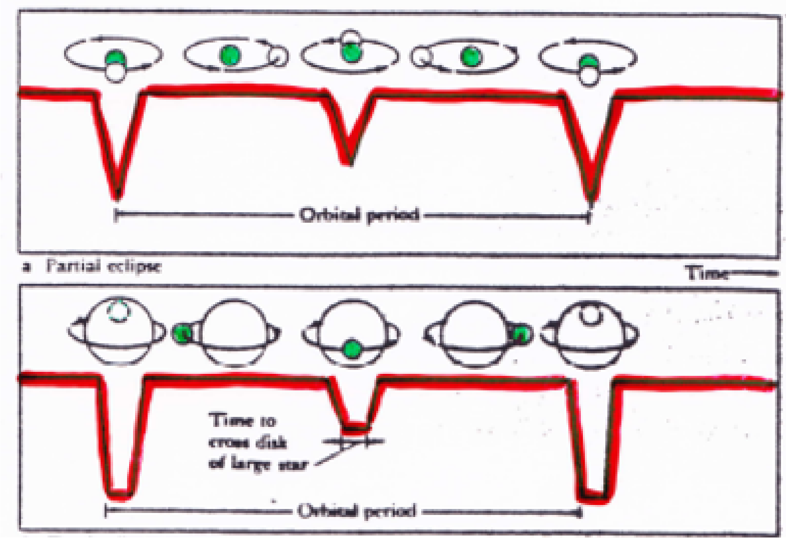


Figure 6.5.1: Partial occultation on top and total occultation on the bottom. Time on the x-axis and luminosity on the y axis.

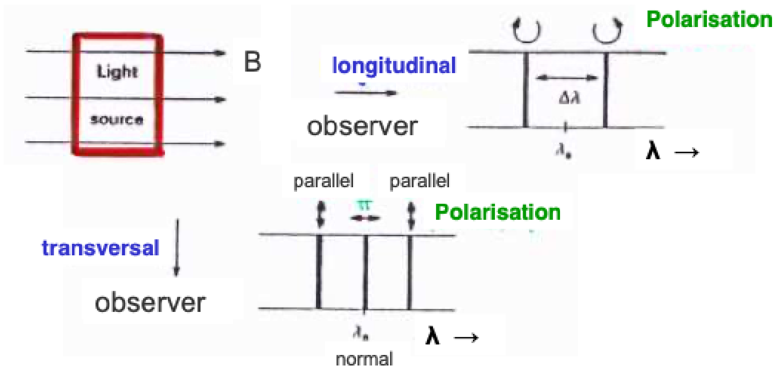


Figure 6.6.1: Zeeman effect

$$D - d = v \cdot (t_3 - t_2) \tag{6.5.2}$$

6.6 Measuring Magnatic fields of stars

The Zeeman effect is a quantummechanical effect that will not be discussed in full depth here, however it is useful for determining the strength of the magnitic field of a star. The Zeeman effect casues spectral lines to split into multiple lines of polarised light and the energy difference $\Delta\lambda$between these Zeeman lines depands on the strength of the magnatic field B.

$$\Delta\lambda = \frac{\lambda^2 gBe}{4\pi m_e c^2} = 4.7 \cdot 10^{-5} \lambda^2 gB \tag{6.6.1}$$

With Lande factor g = ± 1

Chapter 7

Stellar atmospheres

Abstract. Introduction to the different layers of the suns interior and its atmosphere. In doing so, this chapter will also cover the more general radiation transport equation. Lastly it will tackle the suns magnetic field and the effects that come with it.

Keywords: Atmosphere, radiation, magnetic field, flares, CME

Learning goals

- What is the stellar atmosphere, why is it important
- How are they modelled, what is its temperature
- what is limb darkening
- what contributes to the absorption
- which layers constitute the atmosphere
- what is a solar cycle

It is an outermost, tenuous layer, from which the observed radiative flux emerges and/or is it reprocessed.
Stellar interior: Cannot be directly accessed (except neutrinos)
Photosphere: Visible "surface", temperature decreases with height.
Outer atmosphere: Temperature increases again in this region.

Why atmosphere?

pt They are the only source of information we can analyse/measure Quantitative spectrum analysis: "Individuality" of a star based on the abundance of the chemical component it has. We can use it for determining the atmospheric parameters.

Evolutionary state of the stellar classes: Conclude the structure and evolution theory by comparison and analysis of stellar parameters.

Elemental abundance and astro-chemistry, Ideal physics laboratory: Fundamental research in plasma physics, atomic, hydrodynamics, thermodynamics etc.

Stellar atmospheres are geometrically thin (<1 %)

Assumption: 1) Plane parallel approximation (1D), variables such as temperature, particle densities etc depends only on height. 2) Open system: Equilibrium thermodynamics fails but approx.

Modelling atmosphere

Three atmospheric parameters characterises plasma slab (atmosphere):

- Effective temp T_{eff}
- Surface gravity $g = GM/R^2$
- Chemical compositions

The mean intensity at the inner boundary is the Planck spectrum: $F = \sigma T^4$ The plasma slab forms the emerging stellar spectrum from the Planck spectrum at the inner boundary

All interactions between radiation and matters need to be considered.

7.0.1 Modelling the photosphere

Modelling the photosphere based on:

- Intenstiy
- Radiation Transport
- Emerging specific intensity

Radiation Transport Equation

We know from the earlier concepts that we can write relation for intensity as follows:

$$dE = I_\nu cos\theta dAdtd\nu d\Omega \tag{7.0.1}$$

Plane parallel geometry:
θ is the angle between z-axis perpendicular to the geometrical depth (t) and the ray of light that is emerging from the stellar interior.

$$dt = -cos\theta ds \tag{7.0.2}$$

and optical depth is given as: $d\tau = \alpha_\nu ds$ Equation for radiation transport

$$\frac{dI_\nu}{ds} = -\frac{dI_\nu}{dt}cos\theta \tag{7.0.3}$$

$$\frac{dI_\nu}{ds} = J_\nu - \alpha_\nu I_\nu \tag{7.0.4}$$

$$d\tau_\nu = \alpha_\nu dt \rightarrow dt = \frac{d\tau_\nu}{\alpha_\nu} \tag{7.0.5}$$

from the equation 7.0.3 and 7.0.4:

$$-\frac{dI\nu}{dt}cos\theta = J_\nu - \alpha_\nu I_\nu, \tag{7.0.6}$$

which can be brought into the form of the Radiation Transport Equation:

Radiation Transport Equation (RTE)

$$\frac{I_\nu}{d\tau_\nu}cos\theta = I_\nu - \frac{J_\nu}{\alpha_\nu} = I_\nu - S_\nu \tag{7.0.7}$$

Define $z = \tau/cos\theta$ and multiply RTE with e^{-z} and then integrating it over limits from 0 to τ

$$I_{\nu,0} = I_{\nu}e^{-\tau/cos\theta} + \int_0^{\tau} \frac{1}{cos\theta} S_{\nu}e^{-\tau'/cos\theta}d\tau' \tag{7.0.8}$$

- $I_{\nu,0}$: Emerging intensity at the surface of the star
- I_{ν} : Initial intensity at the bottom layer of the star
- $e^{-\tau/cos\theta}$: Extinction in the layer
- $\int_0^{\tau} \frac{1}{cos\theta} S_{\nu}e^{-\tau'/cos\theta}d\tau'$: Radiation part that traverses after the extinction

for semi infinite atmosphere:

$$\tau_{\nu} = \infty : \quad I_{\nu}e^{-\tau/cos\theta} = 0 \rightarrow I_{\nu}(0) = \int_0^{\infty} \frac{1}{cos\theta} S_{\nu}e^{-\tau'/cos\theta}d\tau' \tag{7.0.9}$$

Depending on the source function, optical depth contributes to the radiation but this contribution diminishes by the factor of $e^{-\tau'/cos\theta}$

$$\int_0^{\infty} S \cdot e^{-x}dx = S(x=1) \xrightarrow{I_{\nu}(0,\theta)} = S(\tau = cos(\theta = 0) = 1) \tag{7.0.10}$$

Equation 7.0.10 is Eddington-Barbier Approximation.

Limb darkening

It is an optical effect, where the central part of the star looks brighter than the *limbs or edge* of the star: this is due to the fact that the observer observing the star (Sun) from the Earth is looking vertically downwards making an angle θ with the edge of the Sun, as a result of which an observer sees lower temperature at an optical depth of about 2/3, which appears as though the edges are darker than the body of the Sun. Assuming that the Source function is linear:

$$S_{\nu} = a_{\nu} + b_{\nu}\tau_{\nu} \qquad then: \quad I_{\nu}(0, cos\theta) = a_{\nu} + b_{\nu}cos\theta \tag{7.0.11}$$

$$\begin{cases} \text{if } \theta = 0: & I_{\nu} = a_{\nu} + b_{\nu} \\ \text{if } \theta = \pi/2: & I_{\nu} = a_{\nu} \end{cases}$$

Moments of Specific Intensity

pt To understand the properties of the atmosphere, we compute 'moments of specific intensity' using $cos\theta = \mu$
0th moment: Mean intensity

$$J_{\nu} = \frac{1}{2}\int_{-1}^{+1} I_{\nu}d\mu \tag{7.0.12}$$

1st moment: Eddington Flux

$$H_{\nu} = \frac{1}{2}\int_{-1}^{+1} I_{\nu}\mu d\mu \tag{7.0.13}$$

2nd moment: Radiation pressure

$$K_{\nu} = \frac{1}{2}\int_{-1}^{+1} I_{\nu}\mu^2 d\mu \tag{7.0.14}$$

We know that : $F_{\nu} = I_{\nu}cos\theta d\Omega$

$$\begin{aligned} F_{\tau_{\nu}} &= \int_{\Omega} I_{\tau_{\nu}}\mu d\Omega \\ cos\theta &= \mu \rightarrow d\mu/d\theta = -sin\theta \\ F_{\tau_{\nu}} &= 2\pi\int_{-1}^{+1} I_{\nu}\mu d\mu \end{aligned} \tag{7.0.15}$$

For Plane Parallel atmosphere with $I_\nu(0,\mu) = a_\nu + b_\nu\mu$ and $S_\nu = a_\nu + b_\nu\tau_\nu$

$$\begin{aligned} F_\nu(0) &= 2\pi\int_0^{+1} I_\nu(0)\mu d\mu \\ &= 2\pi\int_0^{+1}(a_\nu + b_\nu\mu)\mu d\mu = \pi(a_\nu + \frac{2}{3}b_\nu) \\ &= \pi S_\nu(\tau_\nu = 2/3) \end{aligned} \tag{7.0.16}$$

Assumptions:

- Thermodynamic Equilibrium: $S_\nu = B_\nu$
- Gray atmosphere $\tau = \tau_\nu$ i.e. extinction has no dependency on wavelength
- Star has a Black body like spectrum with $T = T_{eff}(\tau = 2/3)$

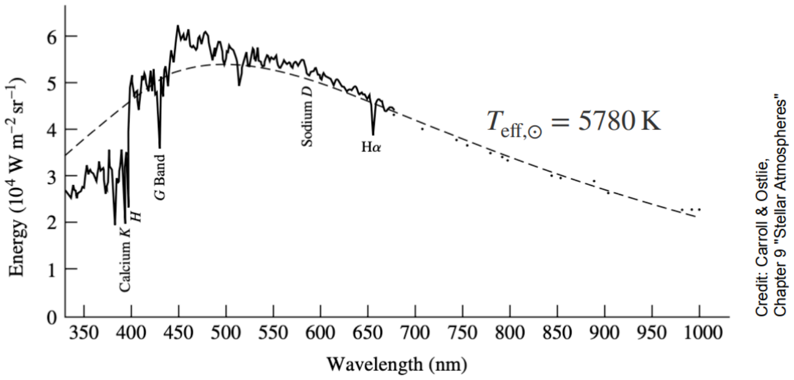


Figure 7.0.1: Measured spectrum of the Sun

Apart from radiative transport, energy transport mechanism includes convection (envelope) and conduction(interior/core) in the star.
Assuming that energy transport is purely due to radiation transport implies that the energy flux remains constant. Each layer of the star can be assumed to be in the local thermodynamic equilibrium.

$$\frac{dF(\tau)}{d\tau} = 0 \rightarrow F = \int_0^\infty F_\nu d\nu = constant \tag{7.0.17}$$

Integrating radiation transport equation and using 0th moment.

$$\frac{dF(\tau)}{d\tau} = 4\pi(J_\nu - S_\nu) = 0 \tag{7.0.18}$$

LTE resembles BB radiation $B(T)$

$$J_\nu = \int B(T)d\nu = \frac{\sigma_{SB}T_{eff}^4}{\pi}\tau \tag{7.0.19}$$

Under assumption that the layer is in LTE with grey atmosphere and isotropic radiation: **Eddington approximation**

Eddington approximation

$$J_\nu = \frac{3}{4}\sigma_{SB}T_{eff}^4(\tau + \frac{2}{3}) \tag{7.0.20}$$

at $\tau = 2/3$: $T = T_{eff}$

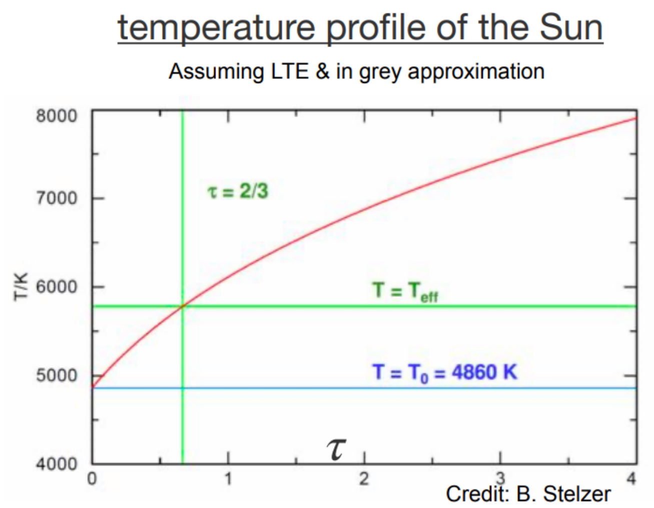


Figure 7.0.2: Temperature profile of the Sun

Absorption coefficient

Assumption: Absorption coefficient is independent of frequency, which is not a realistic assumption, but by considering the mean of the weighted absorption coefficient, result can be approximated: so called **Rosseland mean** True absorption consists of various ways of contribution from processes such as bound-bound, bound-free, free-free, scattering etc.

$$\alpha_\nu = \alpha_{\nu,bb} + \alpha_{\nu,bf} + \alpha_{\nu,ff} + \alpha_{\nu,sc} \tag{7.0.21}$$

Plasma in the star consists of various elements in different states of ionisation. To effectively calculate the absorption coefficient we must know about:

- Population densities of ionisation stages and energy levels
- Cross-section
- Composition of the elements present/metallicity

Statistical equilibrium: For each atomic level, population and depopulation rate is equal.

Bound-free absorption coefficient of Hydrogen

Bound-free α_{bf} involves absorption that leads to the ionisation of an atom

- α_{bf} decreases as the wavelength in the series decreases.
- α_{bf} increases with the increased number of lower energy level (n)
- Edge in the absorption coefficient is produced due to threshold energy that is required for the ionisation.

Unlike in the Grey-atmosphere approximation, characteristic edges are seen in the spectrum as discontinuities in the absorption coefficient.

Metallicity

In astrophysics, every element heavier than He is called as a metal. Metallicity is calculated as a relative abundance of metals in the star compared to the abundance of H and defined relative to the abundance of that metal in the Sun.

$$\left[\frac{X}{H}\right] = \log\frac{X}{H} - \log\frac{X_\odot}{H_\odot} \tag{7.0.22}$$

By definition, Metallicity of the Sun is zero.

- $\left[\frac{X}{H}\right] > 0$ High metallicity
- $\left[\frac{X}{H}\right] < 0$ Low metallicity

7.0.2 Outer atmosphere of the Sun

When we observe the Sun visually, it appears as though there is a very abrupt and clear edge to it, but an actual "surface" does not exist; rather, what we are seeing is a region where the solar atmosphere is OPTICALLY THIN and photons originating from that level travel and travels through space. Since some photons can always escape when the optical depth is greater than unity while others may be absorbed when the optical depth is less than unity but we know the odds of a photon leaving the solar atmosphere diminish rapidly as the optical depth increases.

The Sun's atmosphere changes from being optically thin to optically thick in only about 600 km. This is a relatively small distance (about 0.09 % of the Sun's radius) and this is what gives an "edge" of the Sun its sharp appearance.

Structure

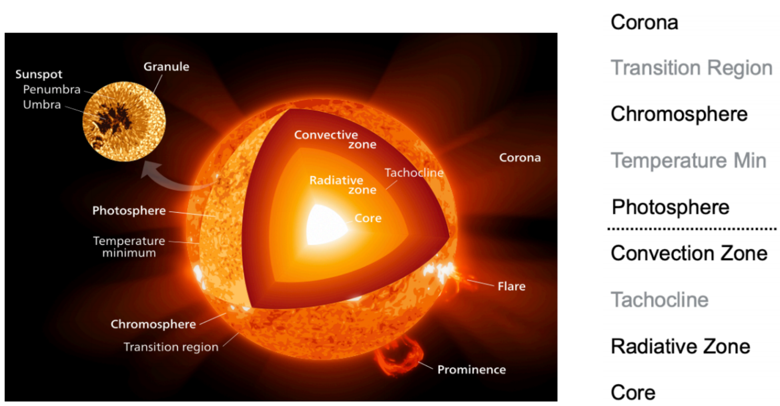


Figure 7.0.3: Different layers of the Sun

Photosphere

The region where optical photon otiginate from is called as the photosphere. The limit for the base of the photosphere is quite arbitrary, hence it is sometimes defined to be 100 km below the level where the optical depth at a wavelength of 500 nm is unity. At this depth, $\tau_{500} \simeq 23.6$ and the temperature is approximately 9400 K Temperature of photosphere decreases as we move outwards, the minimum temperature defines the 'top' of photosphere.

Granulation

Base of photosphere: characterised by the "boiling" patchwork of bright and dark regions from the top of convection zone protruding deep into photosphere. Bright cells are vertically rising hot convective bubbles, energy is released via photons whcih results into cooled, darker gas that sinks back giving the dark outer lining to the granulae. Velocity: 1km/s Brightness: 1 % temperature difference at $\tau = 1 \simeq 500 Kelvin$

Tachocline

Boundary layer between the radiative interior core and conductive outer zone. This is the where differential rotation of the sun converges with the latitude, it is believed that tachocline region is the likely source of the magnetic field of the Sun.

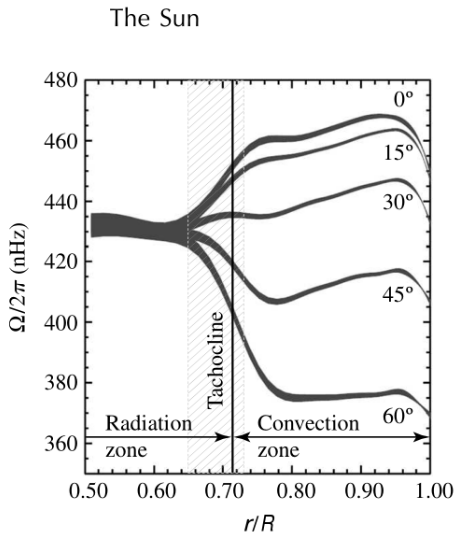


Figure 7.0.4: Differential rotation of the Sun wrt angular frequency

Outer atmospheres

Some important points to note on the outer atmosphere:

- Outer atmosphere: Corona and Chromosphere
- Gases are not in the hydrostatic equilibrium.
- Radiation field do not have local thermodynamic equilibrium
- Magnetic field is not neglectable.
- Spatially and temporally variable emissions take place in the atmosphere
- MHD is coupled with non-LTE radiation transport

Chromosphere

Lies above the photosphere, in this region the hydrogen is completely ionised. Particle density decreasese from 10^{17} to $10^{9cm^{-3}}$ Visible photon emission from this layer is not normally seen due to the bright solar disk, but it can be observed around the limbs at the beginning and towards the end of the total solar eclipse, this is called as **flash spectrum**.

Corona

Corona extends up to the space and does not have well defined boundary, the temperature reaches up to 2 million Kelvin. **Coronal heating problem**: Temperature of the corona is very high and while there is no definite answer, possible explanation lies in the magnetic fields. Possible explanations include, Alfven waves (Transverse plasma waves under MHD) or small scale magnetic reconnection (nano-flares)

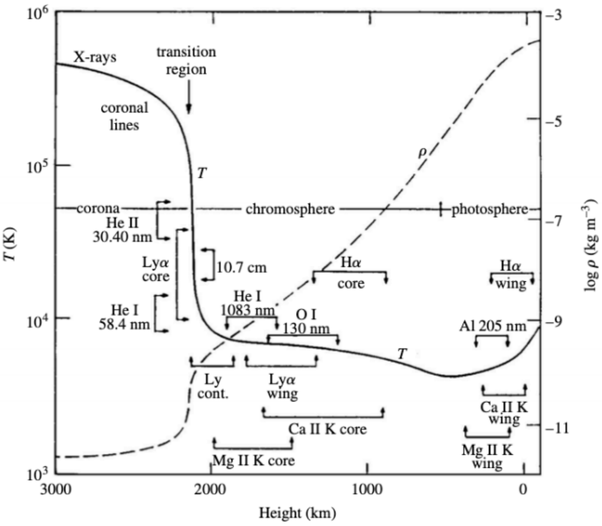


Figure 7.0.5: Temperature and mass density profile of the upper atmosphere of the sun

Solar wind

Stream of charged particle coming from the sun due to different solar activities. Mainly constitute of electron, protons and He nuclei.

- Velocity: $\simeq 800 km/s$ at pole and $\simeq 300 km/s$ at equator
- Density: 5-10 particle per cm^{-3}
- Mass loss due to solar wind, $2-3 \cdot M_{\odot/yr}$

7.0.3 Activity

Sun Spots

Sun spots are dark spots in the optical region with colder compared to the rest of the surface. They are bright in the UV and X ray. They are periodic with minimum and maximum solar cycle that last for about 11 years each. They have strong and active magnetic field which when loops back into the solar interior creates bright arc like structure. Plot of magnetic field with time is known as **butterfly diagramme** because of its wing-like appearance.

Solar Dynamo

Magnetic dynamo theory explains the process of solar dynamo that generates the magnetic field in the sun. Magnetic fields in the sun is axissymetric. According to the theory, the magnetic fields are stored in the gas, with differential rotation of the sun, they are stretched making the poloidal magnetic lines into toroidal known as "wrapping around the sun". This twist leads the magnetic lines to rise to the solar surface, creating the sunspots. Initially, the magnetic "ropes" are formed at higher latitude (Solar minimum) but due to the differential rotation, they move to the lower latitude, creating more sunspots (solar maximum) with opposite polarities.

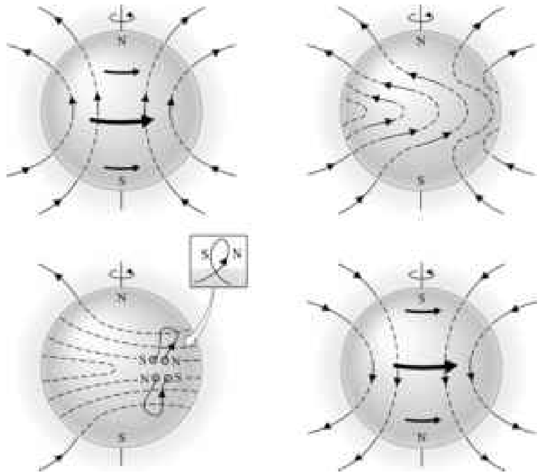


Figure 7.0.6: Magnetic dynamo theory

Coronal holes

Less dense and colder than the corona. Visible as dark regions in X-rays, it evolves with solar cycle. Constitute of open magnetic field through which solar winds escape. Origins of geomagnetic storms during solar minimum cycle.

Solar prominence

Nearly the size of the Earth. Also known as proturberance, filaments. It is a loop of plasma that follows the magnetic field lines. Erupting prominence: Prominence that are unstable and burst outwards, releasing plasma. Lasts from months to days.

Flares and CME

Flares are eruptive event that releases very high energy. CME is the event that releases very highly charged particle into the space.